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# MULTI-STAR TARGET MODEL FOR ASTRONOMICAL ORIENTATION OF THE OLD KINGDOM EGYPTIAN PYRAMIDS 

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During the Pyramid Age, the ancient Egyptians erected some of the most iconic monuments in the world, but their method of alignment and the exact dates of construction remain in dispute. This paper presents new archaeoastronomical evidence that both explains the ostensibly erratic orientation of the Old Kingdom pyramids and offers a novel solution to the dating problem. An analysis of the alignment of pyramids built during the $3^{\text {rd }}$ to $6^{\text {th }}$ Dynasties reveals that they were not oriented to true north, as expected by one of the prevailing current models, but to prominent stars in the northern circumpolar region. A distinct pattern emerges when the time-dependent position of these stars is compared with the orientation of a series of pyramids whose alignments are known. The pattern explains all the available azimuth data of the pyramids from Djoser to Unas and predicts older dates of construction for these structures with an accuracy of no more than five years, up or down. In conclusion, the age of the Old Kingdom is approximately two centuries older than conventionally estimated, according to traditional textual reconstructions of Egyptian chronology. These results are consistent with previous radiocarbon data obtained from samples collected at known Old Kingdom sites

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thereby aligning archaeological physical with archaeoastronomical evidence. The Egyptian chronology serves as a standard reference to establish chronologies in the entire ancient Near East of the $3^{\text {rd }}$ millennium BCE. Therefore, the revised chronology based on the findings presented here warrants a fresh look at the historical timelines of other ancient civilizations contemporary with Ancient Egypt.

Keywords: Old Kingdom pyramids, astronomical orientation, Meskhetiu, imperishable stars, Egyptian chronology, archaeoastronomy, radiocarbon dating

## I. Introduction

Several of the Old Kingdom Egyptian pyramids are oriented to the cardinal points with striking accuracy. This is exemplified by the two big pyramids of Giza - those attributed to Khufu and Khafre, the foundations of which deviate from the meridian line by no more than $3^{\prime}$ and $5^{\prime}$, respectively. These values are near those the human eye can resolve. What method of orientation to the cardinal directions did the pyramid builders use to achieve such impressive results? Over the past century and a half, researchers have proposed various hypothetical methods ${ }^{1}$ to explain this feat. These methods largely fall into two groups:

1) "True north" methods, the accuracy of which depends only on the quality of the observations:

- Observing the elongations of the orbit of a circumpolar star [Petrie 1883, 211-212; Edwards 1947, 209-211]²;
- Observing the meridian transit of a circumpolar star [Romieu 1902, 135-142];
- Observing the shortest shadow produced by a gnomon [Zinner 1931, 1-32];
- Observing the rising and setting position of the sun [Gallo 1998, 77-90].

2) "Precession-susceptible" methods, the results of which contain an additional error because of the precession of the Earth's axis:
[^0]- Observing the rising position of a star [Haack 1984, 119125];
- Observing the vertical alignment of a pair of circumpolar stars [Polák 1952, 219-220; Spence 2000, 320-324; Belmonte 2001, S11-S15];
- Observing the horizontal alignment of a pair of circumpolar stars [Rawlins, Pickering 2001, 699].
The first group of methods was favored by scholars until 1984, when S. Haack [1984, 119-125] discovered that the azimuths of pyramids from the $4^{\text {th }}$ and $5^{\text {th }}$ Dynasties tended to vary with time; "precession-susceptible" methods have been proposed to explain this trend. However, no proposal from either category could explain the entire set of available azimuth data of all those pyramids for which accurate measurements exist. The fundamental weakness of the methods from the "true north" group is that the expected random orientation errors would not form a distinctly systematic trend ${ }^{3}$ (Fig. 1). The methods from the "precession-susceptible" group suffer from exceptions to the rule they attempt to establish, unable to account for data collected from the pyramids of Djedefre, Unas and those from the $3^{\text {rd }}$ Dynasty. Despite significant differences otherwise, both groups of methods have in common the assumption that the goal of the ancient Egyptians was to orient the monuments to true north ${ }^{4}$. Any deviation from due north, consequently, must be the result of ancient surveying errors (first group), or ignoring precession (second group).

[^1]

Fig. 1. Average azimuths of the $4^{\text {th }}-5^{\text {th }}$ Dynasty pyramids. Data from Table 2 (see below).

However, some pyramids' orientations are incompatible with the predictions made within the framework of this key assumption that ancient surveyors were fixated on true north:

1) The pyramid of Teti is rotated by almost $10^{\circ}$ relative to the cardinal directions, while the 30 -year older pyramid of Unas merely one kilometer to the south in the necropolis of Saqqara follows the meridian line almost exactly.
2) The pyramid of the $3^{\text {rd }}$ Dynasty king Djoser deviates circa $3^{\circ}$ from true north. Thus, it represents the first large Egyptian structure known to have been oriented to the cardinal points. However, an explanation is lacking as to why subsequent rulers like Sekhemkhet and Khaba built pyramids that significantly deviate from cardinality by circa $11^{\circ}$ and $8-9^{\circ}$, respectively ${ }^{5}$.

[^2]3) Of the $4^{\text {th }}$ Dynasty pyramids, that of Djedefre deviates the most from cardinality, some 10 times more than the pyramids of Khufu and Khafre ( $47^{\prime}$ vs. $3^{\prime}$ and $5^{\prime}$ ), built before and after, respectively (Fig. 1).
In the first two of the listed cases the deviations in the azimuths from due north are so great that attributing them to observational errors seems incorrect. An alternative explanation - that only some pyramids were oriented to true north while others were aimed at different targets - necessitates a new look at the subject of ancient survey methods to ask if prior models made false assumptions. Therefore, a reexamination of the expanded data set including new azimuth data of Old Kingdom pyramids is needed.

## II. Analyzing the data

To date, all existing $4^{\text {th }}$ Dynasty pyramids have been carefully measured. The three Giza pyramids have been scrutinized the most. The situation is less clear concerning monuments of the $5^{\text {th }}$ Dynasty: azimuth data exist for three of these seven pyramids, while the data for two (Sahure and Neferirkare) are not accurate enough due to the poor state of their bases. For the three pyramids of the $3^{\text {rd }}$ Dynasty, the data have been rounded to whole degrees, as they have never been measured to higher accuracy. Data for the pyramids from the $6^{\text {th }}$ Dynasty are missing. Table 1 shows a compilation of the known azimuths of thirteen Old Kingdom pyramids.

| Pyramid | Azimuth <br> (N side), <br> arcmin. | Azimuth <br> (E side), <br> arcmin. | Azimuth <br> (S side), <br> arcmin. | Azimuth <br> (W side), <br> arcmin. | Azimuth <br> (sides), <br> arcmin. | Azimuth <br> (passage), <br> arcmin. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Djoser $^{6}$ |  |  |  |  | $+180 \pm 30$ |  |
| Sekhemkhet $^{7}$ |  |  |  |  | $-660 \pm 30$ |  |
| Khaba $^{8}$ |  |  |  |  | $-510 \pm 30$ |  |

[^3]| Meidum $^{9}$ | $-35.4 \pm$ | $-20.6 \pm$ | $-23.6 \pm$ | $-18.1 \pm$ |  | $-21.6 \pm$ |
| :--- | :---: | :---: | :---: | :---: | :--- | :---: |
|  | 1.0 | 1.0 | 1.0 | 1.0 |  | 1.0 |
| Bent $^{10}$ | $-07.5 \pm$ | $-17.3 \pm$ | $-04.2 \pm$ | $-11.8 \pm$ |  | $-01.0 \pm$ |
|  | 0.3 | 0.3 | 0.3 | 0.3 |  | 0.3 |
| Red $^{11}$ | - | $-08.7 \pm$ | - | - |  | $+02.9 \pm$ |
|  |  | 0.3 |  |  |  | 0.3 |
| Khufu $^{12}$ | $-03.6 \pm$ | $-03.4 \pm$ | $-00.5 \pm$ | $-03.7 \pm$ |  | $-03.7 \pm$ |
|  | 0.3 | 0.3 | 0.3 | 0.3 |  | 0.3 |
| Djedefre $^{13}$ | $-51.7 \pm$ | $-43.9 \pm$ | $-48.4 \pm$ | $-50.8 \pm$ |  | $?(-20 \ldots-$ |
|  | 1.0 | 1.0 | 1.0 | 1.0 |  | $30)$ |
| Khafre $^{14}$ | $-03.8 \pm$ | $-04.0 \pm$ | $-05.8 \pm$ | $-04.2 \pm$ |  | $-05.6 \pm$ |
|  | 0.3 | 0.3 | 0.3 | 0.3 |  | 0.3 |
| Menkaure $^{15}$ | $+16.8 \pm$ | $+12.4 \pm$ | $+13.0 \pm$ | - |  | $+13.3 \pm$ |
|  | 1.0 | 1.0 | 1.0 |  |  | 1.0 |
| Sahure $^{16}$ |  |  |  |  | $-23 \pm 10$ |  |
| Neferirkare $^{17}{ }^{17}$ |  |  |  |  |  |  |
| Unas $^{18}$ | $+17.8 \pm$ | $+17.1 \pm$ | $+17.5 \pm$ | $+17.4 \pm$ |  |  |
|  | 0.3 | 0.3 | 0.3 | 0.3 |  |  |

Table 1. Azimuth data on the sides and descending passages of the measured Old Kingdom pyramids. Error margins refer to the accuracy of measurements ( $\pm 0.3^{\prime}$ is allowed for recent measurements taken with a meridianseeking theodolite ${ }^{(10,1)} ; \pm 1.0^{\prime}$ - for measurements with a less accurate theodolite ${ }^{(9,13,15)} ; \pm 10^{\prime}$ - for pyramids the orientation of which was calculated from figures in excavation reports ${ }^{(16,17)} ; \pm 30^{\prime}-$ for pyramids the orientation of which was reported rounded to degrees ${ }^{(6,7,8)}$.

[^4]All plausible ${ }^{19}$ orientation methods from the above list imply the use of circumpolar stars to determine the N-S axis of the structure under construction. Therefore, either (a) the East side, or (b) the West side, or (c) the axis of the descending passage would have been initially oriented during an astronomically themed ritual, and, afterwards, all other sides relative to it using geometry. We have no information about which option (a), (b) or (c) took place. Since there are more data from the bases than on the descending passages ( 13 vs. 6 rows in Table 1), the average azimuth of the East and West sides (or the average azimuth of all sides in the absence of specific values) will hereafter be used in all computations ${ }^{20}$.

| Pyramid | Dyn. | Avg. azimuth, <br> arcmin. | Accession date, <br> BCE | Start of construction <br> date, BCE |
| :--- | :---: | :---: | :---: | :---: |
| Djoser | $3^{\text {rd }}$ | $+180 \pm 30$ | $2678[+11]$ | 2677 |
| Sekhemkhet | $3^{\text {rd }}$ | $-660 \pm 30$ | $2659[+11]$ | 2658 |
| Khaba | $3^{\text {rd }}$ | $-510 \pm 30$ | $2651[+11]$ | 2650 |
| Meidum | $4^{\text {th }}$ | $-19.3 \pm 1.0$ | $2624[+11]$ | 2623 |
| Bent | $4^{\text {th }}$ | $-14.6 \pm 0.3$ | - | 2613 |
| Red | $4^{\text {th }}$ | $-08.7 \pm 1.0$ | - | 2604 |
| Khufu | $4^{\text {th }}$ | $-03.6 \pm 0.3$ | 2589 | 2588 |
| Djedefre | $4^{\text {th }}$ | $-47.3 \pm 1.0$ | 2566 | 2565 |

[^5]| Khafre | $4^{\text {th }}$ | $-04.1 \pm 0.3$ | 2558 | 2557 |
| :--- | :---: | :---: | :---: | :---: |
| Menkaure | $4^{\text {th }}$ | $+14.1 \pm 1.0$ | 2532 | 2531 |
| Sahure | $5^{\text {th }}$ | $-23 \pm 10$ | 2487 | 2486 |
| Neferirkare | $5^{\text {th }}$ | $+30 \pm 10$ | 2475 | 2474 |
| Unas | $5^{\text {th }}$ | $+17.3 \pm 0.3$ | 2375 | 2374 |

Table 2. Average azimuths of the E-W sides (or the average azimuth of all sides) of Old Kingdom pyramids and corresponding dates. The accession dates are from Shaw's [2000, 482] chronology ${ }^{21}$ except for the length of Sneferu's reign, for which the middle estimate of 35 years ${ }^{22}$ is used (differences from Shaw's dates are in square brackets). The dates of construction of Sneferu's pyramids are calculated using Stadelmann's proportion ${ }^{23}$. The start of construction date is defined as the year following the date of accession.

Upon initial inspection (Fig. 2), the data belonging to the $3^{\text {rd }}$ Dynasty pyramids appear non-contributory because of the large error margins and could be excluded from the analysis. Nevertheless, these very data are most interesting, as they fail to confirm the expected pattern due to their great azimuthal deviations.

[^6]Multi-star target model for astronomical orientation of the Old Kingdom...


Fig. 2. Average azimuths of Old Kingdom pyramids relative to construction dates. Data from Table 2.

The pyramids examined fall into two groups: a) $3^{\text {rd }}$ Dynasty pyramids with large deviations from true north, and b) $4^{\text {th }}-5^{\text {th }}$ Dynasty pyramids whose orientation deviates from north by no more than 50 arcminutes. The groups are separated by a time interval of a few decades that elapsed between the establishment of the pyramid of Khaba and the pyramid of Meidum. Therefore, either the orientation method employed by Egyptians changed drastically in this relatively short time span, or two entirely different methods were used for the two groups.

The distribution of the azimuth data (Fig. 1, Fig. 2) permits the following conclusions:

1) The trend-forming pyramids, with one exception (Neferirkare), belong to the $4^{\text {th }}$ Dynasty. For reasons unknown, the pyramids of Djedefre and Khafre do not follow the general trend of the $4^{\text {th }}$ Dynasty pyramids.
2) The excellent alignment of the data with the trend line (errors within $\pm 3^{\prime} ;$ Fig. 1) indicates that: a) the orientation method used was both accurate and precise, and $b$ ) the very presence of the azimuth trend serves as an unambiguous indication of the general correctness of the Turin King List data on the sequence of kings and reign lengths for the Old Kingdom. The azimuths would not form such a distinct trend if this royal reign sequence were incorrect.
3) The fact that the data appear to track precession suggests that astronomical observations for each of the trend-forming pyramids were carried out at a specific moment in time corresponding to a short-term recurring celestial event, such as a prominent configuration of two or more celestial objects (for example, their vertical or horizontal alignment).
4) The intersection of the trend line with the $x$-axis indicates that the celestial object used as a reference point at a certain sky position crossed the celestial meridian due to precessional drift a little more than a decade ${ }^{24}$ after the pyramid of Khufu was established.
5) The gradient of the trend line (the angle it forms with the $x$-axis) characterizes the rate of the precessional drift of the reference object. The direction and rate of the precessional drift of circumpolar stars when observed during prominent configurations of two or more celestial objects are not the same for different positions of the celestial sphere. For Egypt, the rate is in the range of circa $\pm 24^{\prime \prime} /$ year. The gradient of the observed trend line of circa $+21^{\prime \prime} /$ year allows us to limit the range of suitable sky positions on which orienting survey ritual of the pyramids could have fixated. All other positions can be excluded from consideration (Fig. 3).

[^7]

Fig. 3. The northern starry sky as seen from the Giza Plateau at the end of the $4^{\text {th }}$ Dynasty. The rate of precessional drift of circumpolar stars is circa $+24^{\prime \prime} / \mathrm{y}$ for the sky position shown (circa $0^{\prime \prime} / \mathrm{y}$ for a $90^{\circ}$ rotation of the celestial sphere; circa $-24^{\prime \prime} / \mathrm{y}$ for a $180^{\circ}$ rotation, etc.). The gradient of the trend line in Fig. 1 limits suitable sky positions to only those when the meridian was inside the gray sectors (light gray sectors below the Pole only), which approximately correspond to the rates of precessional drift of circumpolar stars in the range of $+21^{\prime \prime} / \mathrm{y} \pm 10 \%$. The range of $\pm 10 \%$ accounts for the possibility of minor inaccuracies in the chronological data. Stars are shifting relative to the gray sectors over time due to the precession of the Earth's axis. (Adapted from Stellarium 0.18.2).

Unfortunately, the reference object for the single discovered trend cannot be reasonably chosen, since any prominent star from the gray sectors ${ }^{25}$ could have caused the observed trend in the azimuth data. In the case of a single trend an unequivocal choice cannot be made at all, and any proposal runs the risk of only reflecting subjective biases

[^8]as to orientation method, as to a specific star (or star pair) used as reference point, and as to the absolute dates of the $4^{\text {th }}$ Dynasty. However, the pyramid of Djedefre, cannot be explained with "precessionsusceptible" models as an outlier to the trend line, provides a unique opportunity to solve this mystery. The high-resolution graph (Fig. 4) shows that the azimuths of the pyramids of Djedefre, Sahure and Unas align to make a separate trend ${ }^{26}$ with a gradient similar to that of the main trend. Random similarity of the gradients being unlikely since two (Djedefre, Unas) out of the three pyramids forming the second trend were accurately measured.


Fig. 4. Two groups of pyramids form two trends with virtually identical gradients (about $+21^{\prime \prime} /$ year for the main trend line (Meidum to Neferirkare); about $+20^{\prime \prime} /$ year for the second trend line (Djedefre to Unas)). The data are based on Table 2.
${ }^{26}$ Grigoriev [2015, 2-3] was the first to discover a separate trend, formed by the pyramids of Djedefre, Sahure and Unas: "There are two exceptions: the earliest pyramid of Djoser, with a deviation of about 180' and two later pyramids of Djedefre and Sahure whose deviation from this trend (not from the pole!) is about 50' counterclockwise. The last has been explained by a possible choice of two other stars for orientation. Actually, the later pyramid of Unas, the pharaoh of the $5^{\text {th }}$ dynasty, also gets to the same line with the orientation of these pyramids. [...] Two precession lines are clearly visible: between deviations of the pyramids of Huni and Neferirkare, and the pyramids of Djedefre and Unas".

The presence of two trends indicates that two different stars (or star pairs) were used as reference objects. The similar gradients of the two trend lines suggest that the orientation procedures for both groups of monuments were carried out in the same position of the celestial sphere, since the rates of precessional drift of circumpolar stars in a certain position of the sky differ very slightly. Therefore, Egyptian surveyors aimed for a specific sky position to orient all the pyramids examined belonging to the $4^{\text {th }}-5^{\text {th }}$ Dynasties, except Khafre (see below). Both trend lines intersect the x -axis at time points 149 years apart. This represents the time interval separating two consecutive crossings of the celestial meridian by two different reference objects observed in one and the same position of the sky. Based on azimuth data alone these two reference objects cannot be identified, thus clues must be sought in ancient sources.

To summarize, there are parallel trends in the azimuth data, indicating the existence of a special sky position in which the $4^{\text {th }}-5^{\text {th }}$ Dynasty pyramids were oriented using two different reference objects. The similar gradients of these trends make it possible to focus on a narrow range of suitable sky positions where this might have been.

## III. Special position of the sky

Ancient texts or illustrations that describe the process of orienting Old Kingdom pyramids have not yet been found. Inscriptions carved onto temple walls of later periods indicate that the ancient Egyptians carried out a foundation ceremony called "stretching of the cord" ${ }^{27}$, during which the king and the goddess Seshat ritually set the four corners of the temple based on astronomical observations coupled with unknown manual operations. This ceremony appears to have had a more ancient ${ }^{28}$, possibly pre-dynastic, origin predating these written records by more than two thousand years. No other foundation rituals

[^9]are known yet．Thus，it is possible that an identical，or similar，ritual was used to lay the foundations of both temples and pyramids．An analysis of the＂stretching of the cord＂ceremony is therefore in order．

The most complete descriptions accompanying the images of the foundation ritual are written on Ptolemaic period Egyptian temple walls－the Temple of Horus at Edfu and the Temple of Hathor at Dendera．They contain the following account ${ }^{29}$ ：
無要㘫＊
 m3tj．j r Msht（jw）skj－‘h‘w r－gs mrht．f smn．j hass 4 nw hwt－ntr．$k$
I have taken the pole；I grip the handle of the mallet；I grasp the measuring cord with Seshat．I turn my sight［lit．throw my face］ according to the movement of the stars and I allow my eyes to en－ ter into Meskhetiu．The－one－who－lets－the－lifetime－go－by ${ }^{30}$ is beside his merkhet ${ }^{31}$ ．I establish the 4 corners of your temple ${ }^{32}$ ．
II．


 sbk．n．j m Msht（jw）nwj skj－‘ḩw jp mrht smn．j hass 4 nw hwt－ntrr．k
I have grasped the pole and the handle of the mallet；I grip the measuring cord with Seshat．I observe the course of the move－ ment of the stars．I have seen（the Gods of）Meskhetiu．I am［lit．I belong to］the－one－who－lets－the－lifetime－go－by［who］measures ［with］merkhet．I establish the 4 corners of your temple ${ }^{33}$ ．
${ }^{29}$ Transliteration and translation by the current author．
${ }^{30} s k j-\breve{C}{ }^{〔} w$－epithet of Thoth［Erman，Grapow 1930 （Wb．IV），314，13］； see also［Erman，Grapow 1926 （ $\mathrm{Wb} . \mathrm{I}$ ），222，18）］．
${ }^{31}$ For details about the merkhet see［Borchardt 1899，10－17；Žába 1953， 26－29，56－64；Isler 1991a，53－67］．
${ }^{32}$ Text：［Chassinat 1918，31，Pl．XL d，Tab．Ws． 1 d．III，3］．Translation： ［Žába 1953，58，A a（Pl．II A a）；Brugsch 1880b，622］．Image：［Chassinat 1934，Pl．CCCLXIX］．
${ }^{33}$ Text：［Chassinat 1932，44，Pl．CLXIII d，Tab．J＇o． 1 d．II，2－3］． Translation：［Žába 1953，59，A b（Pl．II A b）；Brugsch 1880b，622－623］．

Multi－star target model for astronomical orientation of the Old Kingdom．．．
III．品 日
dgj $m(p) t r n m t(t){ }^{〔} n h w ~ s j 3$ hns $n M \operatorname{lnt}(j w)$ smn hass nw ḥwt－ntr
［The king］，observing the sky according to the movement of the stars and recognizing the path of Meskhetiu，establishes the cor－ ners of the temple ${ }^{34}$ ．
Two important elements in these passages are relevant here：
－If the ancient observer followed the movement of the stars，as－ tronomical observations were time－consuming．The observer was waiting for a preselected sky position，started the observa－ tions well in advance so as not to miss the right moment．This element confirms the existence of a special position of the sky used to orient the pyramids，predicted by the two trends that emerge from the azimuth data in Fig． 4.
－The object of observation ${ }^{35}$ was the Meskhetiu（Mshtjw）aste－ rism attested with great certainty corresponding to what is to－ day known as the Big Dipper（or Plough）asterism，part of the constellation of Ursa Major［Neugebauer，Parker 1969，183； Parker 1974，60］．During the Old Kingdom，the Big Dipper was a circumpolar asterism at the latitude of Egypt with all its com－ ponent stars visible throughout the year in a configuration de－ pendent on the season and time of day．
${ }^{34}$ Text：［Cauville 2007，90，4－5］．Translation：［Žába 1953，59，C d（Pl．II C c）；Brugsch 1880b，623］．Image：［Cauville 2007，Pl．60］；see also［Dümi－ chen 1877，Pl．L］．
${ }^{35}$ Although the foundation ritual already existed in Early Dynastic times， the mentions of the object of observation known to us belong only to the de－ tailed accompanying texts of the Ptolemaic period，since the Egyptians usu－ ally omitted details and certain scenes in the descriptions［Karkowski 2016， 112］．The obvious symbolic connection of the seven－petalled symbol of the goddess Seshat with the seven stars of Meskhetiu（＂Usually Seshat was por－ trayed with a seven－pointed star（although some have likened it to a seven－ petaled flower）［．．．］It is certain the Egyptians associated the number seven with the Big Dipper because several portrayals of Meskhetiu－at Dendera， Edfu，Esna，and Philae－surround the picture of the bull＇s leg with seven stars．＂［Krupp 1983，25］）may indicate that this asterism was the object of astronomical observations throughout the entire time of the usage of the foundation ritual（the depictions of Seshat＇s symbol date back to the $3^{\text {rd }}$ Dy－ nasty or earlier［Magdolen 2005，197－205］）．

The descriptions given do not mention a particular sky position， nor do they narrow the range of suitable ones，because this circumpo－ lar asterism was visible in all its possible positions．They merely mention that the Big Dipper was used to identify it．This does not sur－ prise in light of the fact that this asterism played an important role in the mortuary beliefs of the dynastic Egyptians［Nemes 2020；Thuault 2020；Arquier 2020］．

However，there are two passages that contain more detail regar－ ding the observations：


［The king］stretches the cord in joy，gives the face m $\ddagger \times 3$ Meskhetiu， and establishes the temple ．．．as before ${ }^{36}$ ．

## V． <br> 


［The king］．．．looks at the sky and sees the stars，gives the face $m$「k〕 Meskhetiu ${ }^{37}$ ．

There are a few interpretations in the literature of the term ${ }^{〔} k 3$ in the above passages：
－In 1877，the German Egyptologist J．Dümichen，when discus－ sing inscription IV，cites the British Egyptologist P．Le Page Renouf＇s interpretation of the related term $r-¢ k 3 i b$ used in the stellar registers of the Ramesside star clocks．Le Page Renouf was the first to suggest that $r$－ $\mathrm{r} / 3 \mathrm{ib}$ denotes a meridian transit or the culmination of a celestial object ${ }^{38}$ ．Dümichen extended this interpretation to the discussed inscription ${ }^{39}$ ．
${ }^{36}$ Text：［Cauville 2007，210，8］．Translation：［Žába 1953，59，C a（Pl．II C a）；Brugsch 1880a，288－289］．Image：［Cauville 2007，Pl．132］；see also ［Dümichen 1877，Pl．XLIV］．
${ }^{37}$ Text：［Chassinat 1928，167，Pl．LXIV，Tab．Cn． 1 g．III，6］．Translation： ［Žába 1953，59，C b（Pl．II C b）］．Image：［Chassinat 1934，Pl．CCCCXXXII］．

38 ＂If the text were Greek instead of Egyptian，there never would have been a doubt as to what was meant by a star being in＇the middle＇．The verb $\mu \varepsilon \sigma o \tilde{v}$, ＇to be in the middle＇，when applied to sun，moon，or star，is equiva－ lent to $\mu \varepsilon \sigma o v \rho \alpha v \varepsilon i v v[\ldots]$ A star is in the middle of its course or in mid－heaven

- In 1880, the German Egyptologist H. Brugsch translated $¢ k>$ of Meskhetiu as culmination of Ursa Major ${ }^{40}$.
- In 1953, the Czech Egyptologist Z. Žába, when discussing inscriptions IV and V, identified $m ‘ \mathrm{k} / \mathrm{s}$ as the compound preposition $m-\ulcorner\mathrm{k} 3$ and translated it as "to/toward" ( $\mathrm{rdj} \mathrm{hr} \mathrm{m}-\mathrm{r} \mathrm{k}$ 3 Mshtjw = turns face to/toward Meskhetiu) ${ }^{41}$.
- In 1983, the American astronomer E. Krupp assumed that ${ }^{〔} k 3$ of Meskhetiu is most likely a particular position of the Big Dipper ${ }^{42}$.
at the moment of its transit or culmination. The technical expression for this in the Egyptian Calendar now before us is er $\bar{a} k[r-c k] ~ i b]$, literally 'in the middle'" [Le Page Renouf 1874, 401-402]. According to the modern point of view, the $r$ - $\mathrm{k} / \mathrm{l}$ ib position in the Ramesside star clocks means "opposite [/in front of] the heart" ( $i b$ - heart, center). Since the observer was sitting facing exactly south, the celestial meridian was accurately in front of him and therefore the star in the $r-C_{k 3}$ ib position was at its culmination. This interpretation is generally accepted to this day [Neugebauer, Parker 1964, ix; Clagett 1995, 61; Leitz 1995, 120; Depuydt 1998, 32]. As for the rest of the positions in the Ramesside star clocks, all $i 3 b j$ positions are passed before the culmination ( $i 3 b j$ - left, $i 3 b t j$ - eastern), and all wnmj positions - after it (wnmj, imn - right, imntj - western). Thus, "left" and "right" are used from the standpoint of the observer, sitting facing south, not the target figure, sitting facing north, as some scholars suggest.

39 "Le Page Renouf gives [...] a very appealing explanation of the term 'er ak', which is not uncommon in the astronomical texts and is also used in the present inscription: [...]" [Dümichen 1877, 30-31].

40 " $\bar{a} q[\mathrm{c} k$ ] ] the true center, in the astronomical sense the culmination of a star or constellation. [...] 'if you notice the culmination of Ursa Major, you mark off the corners of the temple' [Düm. Baugesch. 53]. [...] 'if the face meets the culminating point of Ursa Major, the temple is marked out' [ib.44] The compound prepositions also fall into the concept of 'the middle' [cf. lex. 223]: $m \bar{a} q[m-\ulcorner k]$ ' in the middle of', or 'from the middle of' $[\ldots]$ " [Brugsch 1880a, 288-289]. "The moment of the culmination of Ursa Major observed with the [merkhet ...] was considered to be the traditionally fixed moment for the establishment of the temple building plan laid out in the axis from north to south" [Brugsch 1880b, 623].
${ }^{41}$ See: [Žába 1953, 59, C a , C b, fn. 119].
42 "The texts mention the ak of the Big Dipper, but we don't know what ak means. Most likely it refers to a particular position and orientation of the Plough in its circular course around the Pole". [Krupp 1983, 26].

- In 2001-2008, a group of researchers suggested that ${ }^{6} / 3>$ of Meskhetiu denotes a specific star of the Big Dipper asterism, however, their interpretation is based on a misreading of this term ${ }^{43}$.
It should be noted that $m \times /<>$ in the inscriptions IV and V can be translated in two ways: either a) as the compound preposition $m-\mathrm{r} / 3 \mathrm{or}$ b) as the primary preposition $m$ and the noun ‘ k 3 . Let's look at both cases:
a) The compound preposition $m-\ulcorner/ k$ should be translated as "opposite", "in front of" [Erman, Grapow 1926 (Wb. I), 233, 18-19]. The phrase $r \underline{d} j \mathfrak{h r} m-\lessdot / k 3$ is not attested in the Egyptian texts. Applying this information, we get: "[the king] gives the face opposite Meskhetiu", that is, the observer faces the asterism. Since there is a shorter standard phrase to express the same meaning $r d j \operatorname{lor} r$ (= give face to/toward something), it is not clear why the ancient carver used the unusual phrase with the discussed compound preposition instead of the primary one $-r$.
b) The noun ' $k$ ’ should be translated as "accuracy", "correctness", "straightness" or, possibly, "equality", "level" ${ }^{44}$. The phrase $r \underline{d} j$ $h r m$ is attested in the Egyptian texts with the meaning "pay attention to" [Erman, Grapow 1929 ( $\mathrm{Wb} . \mathrm{III}$ ), 126, 15] ${ }^{45}$. This information gives us the following translation: "[the king] pays attention to ${ }^{46}$ the accuracy of Meskhetiu", that is, the observer notices a certain configuration of the asterism which can be characterized as accurate, straight or, possibly, equal.
${ }^{43}$ Belmonte [2001, S7]; Shaltout, Belmonte [2005, 290-291, Fig. 9]; Miranda, Belmonte, Molinero [2008, 57] and Lull [2008, 92] erroneously read「kJ as $3 h / A x / A k h$ (these are two different terms in Egyptian) and therefore translated it as "spirit", "brilliant", "blessed". Based on the translation the researchers assumed that it could be interpreted as "the brilliant (star)". Since the wrong term was taken into consideration, this interpretation is not valid.

44 "accuracy", "correctness" [Erman, Grapow 1926 (Wb. I), 233, 16]; "straightness" [Faulkner 1991, 50]; "equality", "level" (Z3 sign omitted) [Gardiner 1957, 558]. For $m{ }^{〔} k 3$ as a combination of a preposition and a noun see for example: The Eloquent Peasant, B1, line 130, line 284 [Parkinson 2005, 23, 35]; Kadesh Battle Poem, §153 [Kitchen 1969, 51, §153]; The teaching for Merikare, §128.
${ }^{45}$ For $r \underline{d j}$ hr $m=$ "pay attention to" / "notice" see for example: pBerlin 10024 B, [VS; 5] [Luft 2006, 55]; pBerlin 10037 A-C, [VS; x+21] [Luft 2006, 75].
${ }^{46}$ lit. "gives/puts the face/sight into"; compare with "allow eyes to enter into" in the inscription I.

Based on the texts only, it is impossible to make a definitive choice in favor of one of the two translations. Although the second option is more helpful in the context of this investigation, to date, no information is at hand to know which configuration of Meskhetiu might have seemed accurate, straight or equal to the Egyptians. Since the analysis of the texts accompanying the images of the orientation ceremony yields no further clues, iconographic sources may help.

The earliest known depictions of the Meskhetiu asterism are found on the $9^{\text {th }}-12^{\text {th }}$ Dynasty coffin lids from Asyut [Neugebauer, Parker 1960, Pl. 1-29; Pogo 1932, Pl. A-F] (for example, Fig. 5). The inner side of the lids contain diagrams of the diagonal star clocks, where Meskhetiu is depicted in the company of the three deities - Nut, Sah and Sopdet and appears as a bull's foreleg containing seven stars. The central text field of offerings divides the sky, represented by the inner surface of the lid, into two halves (the northern one with Nut and Meskhetiu, and the southern one with Sah and Sopdet), which are the prototypes of the northern and southern panels making up the vaults of New Kingdom tombs.


Fig. 5. Part of the picture on the inner side of the coffin lid of $J d j$. (S1Tü in the exposition of the Museum of the University of Tübingen; see also [Neugebauer, Parker 1960, Pl. 7-8]). The bull's foreleg is accompanied by the text - Mshtjw m pt mhtt (Meskhetiu in the northern sky).

As can be seen, the deities are rotated 90 degrees clockwise relative to the text, and Meskhetiu aligns with them by standing upright on its hoof ${ }^{47}$. The reason for the turning of the deities toward the head
${ }^{47}$ The orientation of the deities on different lids is the same - toward the head panel. In some cases, they are presented in reverse order, or Nut and Sopdet are mixed up.
panel is not clear, but it could be in observance of the position of the mummy. If we consider both halves as schematic representations of the sky, then the southern one appears unrealistic: Sah should be located in the southern sky, west of Sopdet, not above it. Thus, the relative position and orientation of the deities cannot be regarded as realistic with respect to what was observed in the sky. Therefore, the position of Meskhetiu on the coffin lids does not contain the information needed, and, most likely, is caused by the design features.

Several centuries later, Meskhetiu appears on the astronomical ceilings of New Kingdom tombs (pictures in chronological order: Fig. 6; Fig. 7; Suppl. Materials, Fig. SM1-SM3).


Fig. 6. Part of the northern panel on the ceiling in the tomb of Senenmut (18 ${ }^{\text {th }}$ Dynasty). (After [Wilkinson 1991, Fig. 1]; see also [Pogo 1930, $P l . B-G])$. The picture of a bull is accompanied by the text Mshtjw.

Unlike the coffin lids, in these paintings the deities have different orientations, i.e., vertical, horizontal, and diagonal, thus providing information on the relative positions of asterisms in the sky as the Egyptians imagined them. Meskhetiu is represented in the paintings either as an ovoid bull (Senenmut family; Fig. 6) or a whole bull (Seti I family; Fig. 7). In both traditions it is oriented horizontally in the upper part of the pictures with its head to the left and tip, or tail, to the right.


Fig. 7. Part of the northern panel on the ceiling in the tomb of Seti I in the Valley of the Kings (19 ${ }^{\text {th }}$ Dynasty). (After [Wilkinson 1991, Fig. 2]). The picture of a bull is accompanied by the text Mshtjw.

Comparison of the depictions of the Meskhetiu asterism and two objects associated with it by the Egyptians and used in burial rituals the foreleg of the sacrificial bull and the ceremonial meskhetiu-adze demonstrated an interesting pattern detected by J. Relke and A. Ernest [2003, 64-80, Fig. 5-7, Fig. 9]: a) pictures of the Meskhetiu-bull on the astronomical ceilings depict it in the horizontal position with the tip or tail to the right; b) pictures of the bull's foreleg as a funerary offering to Osiris depict it in the horizontal position with the hoof to the right; c) pictures of the ceremonial meskhetiu-adze in the "opening of the mouth" ceremony mostly depict it in the horizontal position with the handle to the right. In all the listed cases, a bull's foreleg, an ovoid bull, a whole bull, or a ceremonial adze were depicted in a horizontal position with their wide part (head, bowl) to the left, and the tapered part (hoof, tip, tail, handle) to the right. This corresponds to the orientation of the Meskhetiu asterism during the $3^{\text {rd }}-2^{\text {nd }}$ millennium BCE at upper culmination - horizontally above the Pole with the bowl to the left and the handle to the right (see below). Thus, the upper horizontal position of the bull in the New Kingdom paintings is equivalent to the upper horizontal position of the asterism during its upper culmination ${ }^{48}$.

[^10]In general, an asterism is visible best during its upper culmination, and this highest position is uniquely distinct from all other possible ones. Therefore, the ancient Egyptians may have thought of Meskhetiu to be in the most important, possibly sacred ${ }^{49}$ position, which was highlighted in the funerary iconography for this very reason.

It is important that during upper culminations of the Big Dipper in the Old Kingdom epoch, the celestial meridian was inside the right dark gray sector shown in Fig. 3. Consequently, the direction and rate of the precessional drift of stars in this position of the sky were suitable for explaining the observed trends in the azimuth data of the pyramids in Fig. 4. Thus, the iconographic sources allow us to exclude the left dark gray sector in Fig. 3 (as well as the right light gray sector) from the range of candidate sky positions.

However, the culmination of an asterism, as opposed to the culmination of a star, is a long-term event. By contrast, the excellent alignment of the azimuth data with the trend lines indicates that a short-term recurring celestial event must have been used to orient the pyramids. The question arises therefore, which prominent configuration of the stars of Meskhetiu, viewable as a short-term event, took place during its upper culmination and attracted the attention of the Old Kingdom Egyptians? In the Seti I tradition the bull is depicted standing on a horizontal platform (Fig. 7) and seems to be balancing relative to the vertical protrusion, while the whole structure resembles balanced scales ${ }^{50}$. This "balanced" position matches with the abovementioned epithets "accurate", "straight", "equal" or "(scales are)
point, his head tilted sharply forward. The arrangement of the seven stars of the Thigh is known to [...] us from the coffins of Asyut; the development of the Thigh to the ovoid bull (head to the left, tip to the right) in the Senenmut and Ramesseum representation, to the whole bull (head to the left) in the tombs of Seti I and the $20^{\text {th }}$ dynasty leaves no doubt that the left-facing Mascheti [Meskhetiu] is shown in the upper - not in the lower - culmination." [Pogo 1931, 108]. See also [Pogo 1930, 308-311].
${ }^{49}$ Relke, Ernest [2003, 73-74] conclude that the upper position of Meskhetiu was sacred.

50 "[...] The degree of shifting in build-up is even greater in the two other variants from the tomb of Ramesses VI (Ramses VI A/B) belonging to the type appearing in the subgroup of the Seti I A family. Mshtjw appears in both variants in the form of an ox (or a bull) standing on a stripe resembling a stylized balance, [...]" [Nemes 2020, 51].

Multi-star target model for astronomical orientation of the Old Kingdom... level". Significantly, 「 $k 3 j t-\mathrm{a}$ feminine noun with the ‘ $k \gg$ root - is translated as "true balancing" [Faulkner 1991, 50]".

In consequence of the foregoing, what might this "balanced" Big Dipper have looked like in the sky? Due to the elongated shape of this asterism and the location of its outer stars Dubhe ( $\alpha \mathrm{UMa}$ ) and Alkaid ( $\eta \mathrm{UMa}$ ), relative to the Pole in the Old Kingdom epoch (they were roughly at the same distance from it), there was a special position of the asterism during its upper culmination when these two stars were at the same altitude, i.e., were aligned horizontally above the Pole (Fig. 8). In this position, not two, but three brightest stars of the asterism, Dubhe, Alioth, and Alkaid aligned horizontally, and the Big Dipper appeared "balanced" relative to its imagined center, Alioth ( $\varepsilon \mathrm{UMa})$, while the two outer stars represented the balanced scales.


Fig. 8. The "balanced" position of the Big Dipper during its upper culmination in the Old Kingdom sky with the horizontal alignment of the star pair Dubhe ( $\alpha \mathrm{UMa}$ ) - Alkaid ( $\eta \mathrm{UMa}$ ). (Adapted from Stellarium 0.18.2).

This unique, "balanced" position of the Meskhetiu asterism a) fits well with the depictions of the horizontally located bull, foreleg and adze, because at this moment the asterism extended above the Pole strictly along an imaginary horizontal line with its wide part to the left and tapered part to the right; b) explains why the bull in the Seti I tradition (Fig. 7) is depicted balancing on the stylized scales; c) matches well with the epithets "accurate", "straight", "equal" or "(scales are) level" from the descriptions of the orientation ceremony;

[^11]d) is suitable for explaining the azimuth trends in Fig. 4, because during the horizontal establishment of the star pair Dubhe-Alkaid, the celestial meridian was close to the left edge of the right dark gray sector in Fig. 3; e) could be accurately identified in the "stretching of the cord" ceremony using two poles and a cord ${ }^{52}$. There are no other prominent positions of the asterism's stars satisfying the criteria.

To summarize, the analysis of textual and pictorial sources ${ }^{53}$ suggests that the sacred, sought-after position of the sky was the horizontal alignment of Big Dipper's outer stars - Dubhe and Alkaid (Fig. 8). This alignment, to which the "stretching of the cord" rite was targeted, "balanced" the asterism for a short moment, equal to a short-term celestial event. It thus served as a distinct time marker to determine the azimuth of a reference object using the merkhet. Since the merkhet

[^12]Multi-star target model for astronomical orientation of the Old Kingdom... was a small instrument, the target in the orientation procedure was most likely a single circumpolar star. Having determined the exact sky position and reference moment, candidate stars that may explain the two observable azimuth trends can now be identified.

## IV. The synchronism

The two trend lines in Fig. 4 intersect the x-axis. This means that the corresponding reference stars crossed the celestial meridian when observed in the special position of the sky. Since the position of the celestial sphere during the orientation procedure has now become apparent, it is possible to determine the absolute dates when prominent circumpolar stars crossed the celestial meridian in this fixed sky position.

| Star | Apparent <br> magni- <br> tude, $\boldsymbol{m}$ | Year of cros- <br> sing the me- <br> ridian, BCE | Interval between two cros- <br> sings, years |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Alioth | 10 Dra | Pherkad |  |  |
| Thuban ( $\alpha$ Dra) | 3.67 | 2800 | - | 153 | 222 | 484 |
| Alioth $(\varepsilon$ UMa $)$ | 1.76 | 2647 | - | - | 69 | 331 |
| 10 Dra | 4.58 | 2578 | - | - | - | 262 |
| Pherkad $(\gamma$ UMi) | 3.00 | 2316 | - | - | - | - |

Table 3. Complete list of circumpolar stars ( $m<5$ ) that crossed the celestial meridian in the range of $3000-2200 \mathrm{BCE}(2600 \pm 400 \mathrm{BCE})$ when observed in the "balanced" position of Meskhetiu (horizontal alignment of DubheAlkaid). Data from Sky Charts 4.2.1 ${ }^{54}$.

As mentioned above, the two trend lines cross the x -axis 149 years apart. Hence, of all the combinations of star pairs, Thuban and Alioth, which crossed the celestial meridian 153 years apart, are the best fit ${ }^{55}$. The graph in Fig. 9 illustrates the azimuths of the two selected stars, and the others of the Big Dipper, at the "balanced" position of Meskhetiu over time. This graph spans the time range when the two candidate stars precessionally drifted across the celestial meridian.

[^13]

Fig. 9. Data on time-dependent changes in azimuths of the seven stars of the Big Dipper, and Thuban. All azimuth data here, and in the following graphs, correspond to observations at the Giza Plateau. Insignificant differences in the geographical coordinates of the pyramids can be neglected. Data from Sky Charts 4.2.1.

The graphs in Fig. 2 and Fig. 9 can be combined ${ }^{56}$ to match the two trend lines in the pyramids' azimuth data with those two lines that correspond to the precessional drift of the candidate stars Thuban
${ }^{56}$ To combine Fig. 2 and Fig. 9, we must align them so that the intersection points of the two trend lines with the x-axis (149 years apart; see Fig. 4) coincide with the intersection points of the precession lines of the two candidate stars, Thuban and Alioth, with the x-axis (153 years apart; see Table 3). Thus, to obtain the best match, the azimuth data of the pyramids must be shifted 222 years into the past.

Multi-star target model for astronomical orientation of the Old Kingdom... and Alioth (Fig. 10). Since the dates in Fig. 9 are absolute, and the dates in Fig. 2 are relative, depending on the chosen Egyptian chronology, the latter were ignored for this part of the analysis.


Fig. 10. The result of combining the graphs in Fig. 2 and Fig. 9.
The dataset comprising the azimuths of the Old Kingdom pyramids divides into two clearly identifiable subsets: a) the Thuban-oriented group of the $4^{\text {th }}-5^{\text {th }}$ Dynasty pyramids; b) the Big Dipper-oriented group of three $3^{\text {rd }}$ Dynasty pyramids, and three Alioth-oriented pyramids. The detected pattern, involving only five stars, unexpectedly explains all (except Khafre's pyramid; see below) pyramid orientation data known, including the $3^{\text {rd }}$ Dynasty pyramids, whose significant deviations from the cardinal points have been most difficult to explain.

The discovered grouping of azimuths indicates that the stars of Meskhetiu, mentioned in the descriptions of the "stretching of the cord" ceremony, were the targets used to orient the burial complexes since the beginning of the Pyramid Age in the $3{ }^{\text {rd }}$ Dynasty, and the original plan was "one star of Meskhetiu for each one king". From the reign of Sneferu, an innovator in monumental construction, the attention of the Egyptians was fixated on Thuban, and several $4^{\text {th }}$ and $5^{\text {th }}$ Dynasty kings chose for orientation this star again and again; and from the middle of the $4^{\text {th }}$ Dynasty both orientation patterns existed simultaneously, with Djedefre being the first ${ }^{57}$ to return back to the old pattern. The question arises what caused this fixation on Thuban? Since it was the Old Kingdom pole star, due to its proximity to the celestial Pole (Thuban was closer to the Pole than the present pole star, Polaris, is now $-8^{\prime}$ versus $39^{\prime}$ ), it occupied the position of a "central star", around which all other stars wander. The ancient priests could perceive the "motionless" central star as the only place that allows the soul of a deceased king to safely "moor"58 to the rotating firmament and be adopted in the sky among the stars.

In any case, Thuban is the best reference star for the main trend pyramids, since: (a) its proximity to the Pole would significantly reduce the influence of instrumental errors during the orientation ritual [Puchkov 2019, 23, Table 2]; (b) the rise and fall of the construction of the big pyramids can be attributed to its slow drift to and away from the Pole [Puchkov 2019, 57, Fig. 37].
${ }^{57}$ Thuban began to move away from the celestial Pole, losing its status of the central star, during Djedefre's reign. Perhaps this caused disappointment in the pole star cult (Djedefre returned back to the old orientation pattern) and strengthening of the solar cult (he introduced the royal title "Son of $R e$ ").
${ }^{58}$ On the vaults of the New Kingdom tombs, Meskhetiu is depicted tied by a chain to the Mooring-post (Fig. 7, Fig. SM1), or a triangle next to it (Fig. 6). This chain, the celestial analogue of a cattle leash, caused Meskhetiu to move (rotate) in the northern sky, therefore the Mooring-post must correspond to the center of rotation of the Old Kingdom sky (see [Locher 1985, S153; Polák 1952, 177-178, Fig. 7a]). Thus, it can be assumed that the "motionless" Thuban was perceived by the Egyptians as (top of?) the immovable Great Mooring-post (mnjt wrt: §863b [PT 458], §872b-c [PT 461], §884b [PT 466], §1366a [PT 553]), to which the soul of the deceased king should "moor". The triangle in the Senenmut family (Fig. 6) probably corresponds to the location of the celestial Pole during the New Kingdom.

Since several stars are included in the pattern proposed here, we need to check its uniqueness and confirm its validity on an extended input dataset. To begin with, it is necessary to check if the azimuth data of the pyramids correspond to the azimuth data of the stars in the historically expected period. According to the data in Table 3, the reference star for the main trend pyramids in this period would be either Alioth or 10 Dra. However, verification did not reveal any match for the second trend, or the three $3^{\text {rd }}$ Dynasty pyramids. Continuing, it is necessary to confirm the proposed pattern using the new azimuth data of the Old Kingdom pyramids as control data (Fig. 11). The only pyramid available to test in this regard is that of $6^{\text {th }}$ Dynasty king Teti. This pyramid in North Saqqara has an inexplicably large deviation to the west of north, although the neighboring pyramids of Userkaf, Unas, and Djoser, are more well oriented relative to the cardinal points. No accurate azimuth data for the pyramid of Teti have been obtained, but data for the pyramid temple, usually co-aligned with the pyramid, are available, measured to be $-9.25 \pm 0.5^{\circ}$ [Shaltout, Belmonte, Fekri 2007a, 145-146, Table 1].


Fig. 11. Adding data on the pyramid temple of Teti in Saqqara. Although the temple may be inaccurately co-aligned with the pyramid, it can be assumed that
the pyramid of Teti was oriented toward Phecda ( $\gamma \mathrm{UMa}$ ), like the pyramid of Sekhemkhet nearby. All three trend lines have similar gradients ( $+21^{\prime \prime} /$ year main; $+20^{\prime \prime} /$ year - second; $+20^{\prime \prime} /$ year - third). The inaccurate matching of the gradients may be due to some inaccuracies in pyramids' azimuths (Djoser, Menkaure) and reign lengths (see Suppl. Materials, Table SM2, Fig. SM5SM6 for how different estimates of the reign lengths affect the regularity).


Fig. 12. Combination of the azimuth data of the pyramids and stars for the horizontal alignment of the star pair Dubhe-Mizar. Data from Sky Charts 4.2.1.

Finally, we have to examine how the accuracy of determining the sacred position of the sky during the orientation ceremony would have affected the observed pattern. Six minutes after the "balanced" position of Meskhetiu (horizontal alignment of Dubhe-Alkaid), its two lower components, Dubhe and Mizar, were aligned horizontally in the Old Kingdom sky. We can check out this position by creating a new graph (Fig. 12). Only six minutes of discrepancy from "true balance" of the asterism strongly affected the position of its seven stars, because they were situated far from the celestial Pole. Therefore, their

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precessional lines would have significantly shifted, relative to the precessional line of Thuban. Because of this, there would have been no longer a correspondence between Alioth and the pyramids of Djedefre, Sahure and Unas; Phecda, as well, no longer would fit with the pyramid of Sekhemkhet.

The test shows that: a) the match is good for all chronology reconstructions that do not deviate much from the Turin King List data on the reign lengths of the Old Kingdom (see Suppl. Materials, Table SM2, Fig. SM5-SM6); b) neither consideration of other time ranges, nor any other positions of the Big Dipper within the permissible range allows us to find the reference stars simultaneously for two (three?) trends and single data on the $3{ }^{\text {rd }}$ Dynasty pyramids. Thus, it can be stated that only the "balanced" position of Meskhetiu and only specific absolute dates provide an explanation for the orientation of the twelve Old Kingdom pyramids, therefore the pattern found is unique.

The following conclusions can be drawn at this stage:

1) The discovery of azimuth trends with similar gradients (Fig. 11) indicates that the pyramids were oriented toward different stars in the same position of the sky.
2) The accuracy of determining the direction in the orientation rites of the $4^{\text {th }}-5^{\text {th }}$ Dynasties was approximately constant and amounted from 1-2' for Thuban (due to the low rate of change in the pole star's azimuth) up to 3-7' for Alioth (see Suppl. Materials, Table SM1).
3) The Old Kingdom pyramids, in accordance with the religious beliefs of that time, were oriented to selected circumpolar ("imperishable") stars, which were perceived as the place of the king's afterlife, while these stars were the goals themselves, and were not used as supporting markers for orientation to an invisible abstraction, that is, to the celestial Pole ${ }^{59}$.

[^14]4) The hypothesis about the orientation of the $4^{\text {th }}$ Dynasty pyramids to true north is a mistake that arose due to the proximity of the direction to true north and the direction to one of the stars (the pole star) chosen by the Egyptians to orient the pyramids.
Analyzing the grouping of the pyramids' azimuths, we found the unique pattern that explains the orientation of a large heterogeneous group of pyramids, but contradicts the expectations of the conventional chronology about the dates of the Old Kingdom. In order to reconcile star dates and dates based on written records, a discrepancy of circa two centuries must be explained, or the pattern emerging from this analysis must be discarded as a random coincidence.

## V. A brief study of the Egyptian chronology

Knowledge of Egyptian chronology is based on information from Egyptian King Lists, which are either complete but contain significant gaps in the text (Turin King List), or initially incomplete (Abydos King List); and Manetho's figures known to us from later citations, which often differ in detail. Actual reconstructions of Egyptian chronology [Beckerath 1997, 187-192; Shaw 2000, 481-489; Hornung, Krauss and Warburton 2006, 490-495] combine relative data on the sequence and duration of the kings' reigns with the basis of the Sothic dates, which can be converted to absolute dates with reasonable accuracy. This approach implies that the periods of Egyptian history are always associated with some uncertainty, which increases with deepening into the past and moving away from the Sothic "anchor points", so the dates that reconstructions of the Egyptian chronology offer us are estimates.

Information on the sequence of kings and reign lengths of both the Middle and Old Kingdoms is sufficient to roughly reconstruct their chronological structure (relative dates). Unlike the Middle Kingdom, which is pinned to absolute dates more or less accurately due to the Sothic date from the Illahun archive [Parker 1977, 177-184; Rose 1994, 237-261; Krauss 2006b, 448-450], the absolute dates of the Old Kingdom are very approximate because of the significant uncertainty in the duration of the $9^{\text {th }}-10^{\text {th }}$ Herakleopolitan Dynasties before the beginning of the $11^{\text {th }}$ Theban Dynasty ${ }^{60}$. Estimates of the

[^15]Multi-star target model for astronomical orientation of the Old Kingdom...
length of this period range from conventional $0-50$ years $^{61}$ up to one or two centuries according to Manetho. All chronologists agree that the $9^{\text {th }}-10^{\text {th }}$ Dynasties present a source of uncertainty due to insufficient information as to their length (Table 4). Only Manetho reports the duration of the Herakleopolitan rule, while Eusebius and Africanus diverge significantly in estimating the value, providing a weak basis for chronological reconstructions.

|  |  | Manetho (Africanus) | Manetho (Eusebius) | Turin <br> King <br> List | Karnak King List | Abydos King List | Saqqara Tablet |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 9^{\text {th }} \\ \text { Dyn. } \end{gathered}$ | Number of kings | 19 | 4 | - | - | - | - |
|  | Duration | 409 | 100 | - | - | - | - |
| $\begin{aligned} & 10^{\text {th }} \\ & \text { Dyn. } \end{aligned}$ | Number of | 19 | 19 | - | - | - | - |
|  | Duration | 185 | 185 | - | - | - | - |
| Total number of kings |  | 38 | 23 | 18 | - | - | - |
| Total duration |  | 594 | 285 | (lost) | - | - | - |

Table 4. Data on the number of kings and the duration of the $9^{\text {th }}-10^{\text {th }}$ Dynasties according to ancient sources.

There are two models of the First Intermediate Period (FIP), reflecting two different views of this period: a "short model" based on the arguments of the chronologists, and a "long model" followed Manetho's data. The conventional "short model" of the FIP - and, consequently, conventional dates of the Old Kingdom and all earlier periods - is based on the following three rationales:

1) J. Málek suggests that the division of a single line of the Herakleopolitan kings into two separate dynasties could have occurred because of a misunderstanding when copying chronological documents ${ }^{62}$. If this is the case, then Manetho's $9^{\text {th }} \mathrm{Dy}$ nasty turns out to be fictitious and should be ignored.
${ }^{61} 0-50$ years [Beckerath 1997, 188]; 35 years [Shaw 2000, 483]; 38 years [Hornung et al. 2006, 491].
${ }^{62}$ Málek [1982] reconstructed the structure of the "master copy" from which the Turin King List was copied. It turned out that Manetho's (Eusebius's) data on the $9^{\text {th }}-10^{\text {th }}$ Dynasties can be explained by the division of a single line of kings between two columns of the original document:
2) J. von Beckerath reduced Manetho's 185 years for the $10^{\text {th }} \mathrm{Dy}$ nasty to $100-150$ years due to the small quantity of archaeological finds datable to this period [Beckerath 1997, 144].
3) Evidence suggests that the kings of the $10^{\text {th }}$ and $11^{\text {th }}$ Dynasties ruled in parallel for some time over different parts of the country. The length of the parallel reign is estimated from 87 to 114 years ( 100 years on average), hence, the duration of the Herakleopolitan Dynasty before the beginning of the Theban kingdom is reduced by an average of 100 years ${ }^{63}$.
Of the three rationales, the third is convincing; the first seems disputable ${ }^{64}$; and the second is clearly unreliable, as S. Seidlmayer [2006, 165] rightly pointed out, sparse material heritage, in itself, cannot be unequivocal proof of the short duration of the period. If we adjust Manetho's figures according to the first and third arguments, then the Herakleopolitans could have ruled up to a full century before
a) 4 king names in Col. 7 correspond to 4 Eusebius's kings of the $9^{\text {th }}$ Dynasty; b) a total of 18 king names in Col. 7 and Col. 8 correspond to 19 Eusebius's kings of the $10^{\text {th }}$ Dynasty. If so, the division of a single line of 18 kings into two dynasties is associated with a shift in the column and 4 kings in Col. 7 are counted twice. As for Africanus's data, the number of the $9^{\text {th }}$ Dynasty kings duplicates the number of the $10^{\text {th }}$ Dynasty kings.
${ }^{63}$ The reunification of Egypt took place between the $14^{\text {th }}$ and $41^{\text {st }}$ years of Mentuhotep II, the $5^{\text {th }}$ king of the $11^{\text {th }}$ Dynasty. Although the lengths of the individual reigns of the first 4 kings are partially lost, their sum can be estimated at about 73 years, therefore, it took from 87 to 114 years from the beginning of the Theban Dynasty to the defeat of the Herakleopolitans (see [Seidlmayer 2006, 160-162, 165]). Given the rule of the first 5 Thebans for about a century, the assumption of a succession of the 18 Herakleopolitans over 120-130 years looks like an underestimation of the length of their rule.
${ }^{64}$ There are other explanations for the division of the Herakleopolitans into two separate dynasties. For example, Seidlmayer [1997, 85] suggests that the division may have been caused by the political situation, i.e., the rise of the $11^{\text {th }}$ Dynasty. Ryholt [2004, 146, fn. 56] finds Málek's argument not entirely consistent: "[...] if Manetho failed to realize that a shift in column did not necessarily indicate a dynastic shift, one would expect that he would also have cut into segments the other dynasties that happened to be carried over from one column to another, especially numerous kings belonging to the $13^{\text {th }}$ and $14^{\text {th }}$ Dynasties, but this is evidently not the case". 30 years), and there is substantial evidence ${ }^{65}$ to support this.

It should also be added that to eliminate uncertainty of a chronology under study, synchronisms with other chronologies and astronomical (lunar and stellar) synchronisms are used. The earliest cogent synchronism ${ }^{66}$ in Egyptian history is dated back to the $18^{\text {th }}$ century BCE and thus took place after the period of interest to us. The earliest ${ }^{67}$ more or less accurately dated astronomical event is

[^16]the above-mentioned observation of the heliacal rise of Sirius, dated back to the reign of Senusret III of the $12^{\text {th }}$ Dynasty. All earlier periods of Egyptian history have no synchronisms and, therefore, the dates for them are calculated solely by summing up the duration of the reigns from the incomplete and inconsistent ancient royal lists, corrected on the basis of the attested inscriptional dates of any particular king. Thus, the Old Kingdom still remains chronologically unresolved, with conventional chronology favoring the lower end of the possible date range.

Given the above, it can be concluded that there are no compelling arguments against a substantial increase of the length of the FIP, and its longer duration is confirmed by Manetho's data on the $10^{\text {th }}$ Dynasty and tangible evidence. But the difference of a little more than one century with the predictions of the proposed model remains, and at the moment it is difficult to explain it using material or textual evidence. However, the results of radiocarbon dating (see the next section) indicate that all discrepancies can be attributed to an underestimation of the duration of the FIP, so Manetho's figure for the total length of the Herakleopolitan rule may be of historical value.

## VI. A brief study of the results of radiocarbon dating

At the turn of the millennium, there was a large-scale study by Bonani et al. [2001, 1297-1320] that demonstrated an underestimation of the age of most Old Kingdom structures in the conventional chronology by about two centuries. An unprecedented number of 450 samples have been taken from sites dating from the Early Dynastic and low chronologies respectively (see [Depuydt 2000, 172-184]). It is assumed that the dates of the moveable $w 3 g$ feast were determined by the lunar calendar and took place on the $18^{\text {th }}$ lunar day, therefore, the authors assume the latest possible heliacal rise of Sirius at III 3ht 10, which gives about 2495 BCE [arcus visionis $9-10^{\circ}$ ] for Neferefre's Year 1 in their minimum high chronology. But this implies the celebration of the moveable $w 3 g$ in the first lunar month after the heliacal rising of Sirius, although data from the Illahun archive indicate that it was celebrated in the second month [Krauss 1985, 86-94; Luft 1994, 41]. If, by analogy with Illahun, it took place in the second month, then the resulting chronology must be shifted into the past by 120 years or more, depending on the rule (unclear to us) for establishing the date of the moveable $w 3 g$ during the OK .

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Period to the Middle Kingdom. The authors combined their results in the graph (Fig. 13) where all calibrated date ranges derived from the weighted average radiocarbon age of each sample set were compared to the Clayton's chronology, and published the details of the 271 samples in the appendix to the report.


Fig. 13. Comparison of the calibrated ${ }^{14} \mathrm{C}$ ranges ( $1 \sigma$; IntCal98; black bars) with the chronology of Clayton [1994] (hatched areas). (After [Bonani et al. 2001, Fig. 1]).

Although this study mainly analyzed charcoal, for which the inbuilt age of wood (growth age plus storage age ${ }^{68}$ ) must be taken into account, three important details deserve attention:

1) Mixed results for simultaneous (temple and pyramid of Userkaf;

Fig. 13, monuments 19 and 20) or successive buildings ( $1^{\text {st }} \mathrm{Dy}$ nasty tombs; Fig. 13, monuments 1-5) with small associated
${ }^{68}$ To estimate the correct age of the context, when analyzing wood, the following corrections should be taken into account: a) the time difference between the end of life of the sample and its usage (between the felling of trees and their usage in construction) - storage age [McFadgen 1982, 384; Waterbolk 1971; McFadgen et al. 1994, 223]; b) the age difference between the inner and outer rings of a tree - growth age [McFadgen 1982, 384]; c) the possibility of reusing materials that could be taken from earlier buildings [Lehner et al. 1999, 33; Manning 2006, 341].
sample sets indicate the need to analyze larger sets to reduce the influence of hard-to-identify irrelevant material (e.g., building/restoration activity in short time after the original construction event; reuse of some wood; etc.).
2) The age of most monuments is so much older than expected (in many cases, the expected dates are far outside the calibrated ranges for the large sample sets), that discrepancies cannot be attributed to the inbuilt age of wood (see below).
3) Both $12^{\text {th }}$ Dynasty pyramids (Senusret II, Amenemhet III; Fig. 13, monuments 29 and 30) produce similar results that are in good agreement with the expected dates, that is, after the problematic $9^{\text {th }}-10^{\text {th }}$ Dynasties (see previous section), the results coincide well with expectations. A securely dated Sothic "anchor" for the Middle Kingdom exists, and for the pyramids of this period the results agree with expectations, while lacking any anchors for earlier monuments, for which the results and expectations do not match.
In 2009 Dee et al. re-analyzed Bonani's et al. data and modelled the end boundaries (completion date of construction) to estimate the age of the $4^{\text {th }}$ Dynasty, the most problematic in the original publication. This approach, which was designed to take into account the inbuilt age of wood and the own age of the monuments, shifted the results to the expected dates. According to the authors' logic, the average inbuilt age of wood is equal to the difference between the average age of the sample set and the date of completion of the corresponding building. Having modelled $4^{\text {th }}$ Dynasty dates close to those expected, the authors did not report the average inbuilt age of wood for their model output. Here, they are listed:

| $\begin{aligned} & \text { Pyramid of } \\ & \text { the } \\ & 4^{\text {th }} \text { Dynasty } \end{aligned}$ | Avg. age of samples ${ }^{69}$, ${ }^{14} \mathrm{C}$ BP | Number of samples | End boundary estimate $^{70}$, cal BCE (1 $\boldsymbol{\sigma})$ | End boundary estimate, ${ }^{14} \mathrm{C}$ BP | Avg. inbuilt age of wood, ${ }^{14} \mathrm{C}$ years |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Meidum | $4110 \pm 23$ | 7 | 2609-2533 | $4065 \pm 23$ | 45 |
| Bent | $4133 \pm 41$ | 2 | 2618-2530 | $4050 \pm 41$ | 83 |
| Khufu | $4157 \pm 10$ | 44 | 2559-2518 | $4045 \pm 10$ | 112 |

[^17]Multi-star target model for astronomical orientation of the Old Kingdom...

| Djedefre | $4229 \pm 22$ | 11 | $2550-2497$ | $4025 \pm 22$ | 204 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Khafre | $4173 \pm 13$ | 24 | $2527-2463$ | $3975 \pm 13$ | 198 |
| Menkaure | $4127 \pm 11$ | 30 | $2456-2370$ | $3940 \pm 11$ | 187 |

Table 5. The average inbuilt age of wood for Dee's et al. [2009] model output. The ${ }^{14} \mathrm{C}$ age of each end boundary has been estimated in Calib 8.1.0 by using corresponding calibrated date range.

The average inbuilt age of wood for the largest sample sets (pyramids of Giza) reaches values from 100 to $200{ }^{14} \mathrm{C}$ years. It is difficult to accept that materials of such significant average inbuilt age could be used for all these monuments (although the ${ }^{14} \mathrm{C}$ years do not correspond exactly to the calendar years, this would nevertheless suggest the construction of the pyramids of Giza mostly using centuries-old trees). The underestimation of the age of the $4^{\text {th }}$ Dynasty buildings in the study becomes clear. Thus, the end boundary approach disguised, rather than solved, the "old wood" problem by moving the excess of the age of the samples into the inbuilt age of wood.

There is also a study by Bronk Ramsey et al. [2010], which is based on short-lived plant remains such as seeds, plant-based textiles, plant stems, etc. taken from museum collections ${ }^{71}$. Although the chronology modelled by the team is consistent with the conventional chronology, the dating of the Old Kingdom due to the significant shortage ${ }^{72}$ of samples from this period is of limited value. The authors did not include Bonani's et al. data on short-lived materials (37 OK samples in total), and used only their own small dataset with most of the dates belonging to Djoser[/Khasekhemwy]. They also introduced information on reign lengths (except for the FIP) and created a combined model for the Old and Middle Kingdoms (see the OxCal code in their Table S5) due to uncertainty in the length of the FIP.

When analyzing this study, attention should be paid to the fact that most of the Old Kingdom dates (11 out of 13, outliers excluded) belong to Djoser[/Khasekhemwy] (9 dates) and Sneferu (2 dates), for

[^18]which the discrepancies between the results and expectations are minimal (see Fig. 13). At the same time, the most problematic period, including the reigns of Khufu, Djedefre, Khafre, Menkaure, whose monuments have the largest deviations according to Bonani et al., is not represented in the Oxford model. Thus, this chronological model of the Old Kingdom, based on an incomplete dataset does not rest on a firm foundation.

Comparison of Bonani's et al. and Bronk Ramsey's et al. data on short-lived samples, show that the data are variable and therefore two opposite models can be created on their basis. The Oxford model, which supports the conventional Old Kingdom dates and the "short model" of the FIP, is created on several radiocarbon dates matching expectations. In contrast to the Oxford model, the "older model", which supports Manetho's "long model" of the FIP, and older dates of the Old Kingdom, can be built on data of the short-lived materials by Bonani et al. (Sekhemkhet - 2 dates, Djedefre -7 dates, Shepseskaf - 3 dates, Teti - 6 dates; see Table 6). While the Oxford model largely ignores odd, post-Sneferu dates (single dates for Djedkare and Pepi I are not sufficient), Djoser's dates in the framework of the "older model" can be explained by the peculiarities of the collection ${ }^{73}$ and analysis ${ }^{74}$ of the samples.

Since then, there have been no large-scale studies on radiocarbon dating of the Old Kingdom. The review of the main studies reveals that the collection of Old Kingdom radiocarbon data consists mainly of Bonani's et al. charcoal dates (189), the majority of which are so much older than expected that they cannot be explained by the inbuilt age of wood. Modern radiocarbonists tend to dismiss Bonani's et al. data and have focused on a few dates (17 [4 outliers]) from the Ox-

[^19]Multi-star target model for astronomical orientation of the Old Kingdom... ford model, now considered standard. These dates are less than a third of the dataset of short-lived samples from the $3^{\text {rd }}-8^{\text {th }}$ Dynasties (only Bonani et al. and Bronk Ramsey et al. give a total of 54 dates), but they are favored because of their consistency with the generally accepted age estimates for this period.

Below (Table 6) is a comparison of the proposed model for the pyramids of the $3^{\text {rd }}-6^{\text {th }}$ Dynasties with their calibrated date ranges derived from Bonani's et al. short-lived samples.

| Pyramid | $\begin{gathered} \text { Average } \\ \text { age, }{ }^{14} \mathbf{C} \\ \text { BP } \end{gathered}$ | Number of shortlived samples ${ }^{75}$ | Calibrated date range, BCE |  |  |  | Start of construction date by multi-star target model, BCE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 16 (68\%) |  | 26 (95\%) |  |  |
|  |  |  | from | to | from | to |  |
| Djoser | $4120 \pm 25$ | 5 | 2850 | 2625 | 2866 | 2578 | $2899 \pm$ |
| Sekhemkhet | $4217 \pm 58$ | 2 | 2902 | 2696 | 2918 | 2587 | $2880 \pm 5$ |
| Djedefre | $4169 \pm 26$ | 7 | 2875 | 2697 | 2881 | 2633 | $2787 \pm 5$ |
| Shepseskaf | $4209 \pm 35$ | 3 | 2890 | 2703 | 2901 | 2670 | $2724 \pm 5$ |
| Teti | $4111 \pm 21$ | 6 | 2846 | 2583 | 2859 | 2577 | $2566 \pm 5$ |

Table 6. Comparison of the calibrated ranges (Calib 8.1.0) with the expectations of the proposed model.

The proposed model predicts ranges that fall within the 1 -sigma calibrated date ranges in three out of five cases. The data reveal minor internal inconsistencies, and expectations for the monuments of Djoser and Teti are slightly outside the ranges (higher and lower, respectively), likely due to an insufficient number of associated samples. The following graphs summarize astronomical, radiocarbon ( $1 \sigma$ ranges), and historical estimates for the sites corresponding to the two largest sets in Table 6, Djedefre (Fig. 14a) and Teti (Fig. 14b).

[^20]

Fig. 14 (a, b). Comparison of historical, radiocarbon and astronomical estimates of the construction dates of the (a) Djedefre and (b) Teti burial complexes. The average radiocarbon age of the monument corresponds to several calibrated date ranges with different probabilities. Belmonte's hypothesis has $95 \%$ probability for simplification of the display of overlapping ranges.

The graphs show that in both cases, the historical estimates are far from the calibrated date ranges, while the predictions of the proposed multi-star target model fit them much better. The date matches indicate that the data on the radiocarbon age of the Old Kingdom

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monuments are compatible ${ }^{76}$ with the predictions of the proposed hypothesis, and that the chronological discrepancies for this period can be attributed to an underestimation of the duration of the FIP.

To summarize the last two sections, then, there is an uncertainty about the age of the Old Kingdom by circa 70 years (up to 170 years if the $9^{\text {th }}$ Dynasty is real) due to different estimates of the duration of the FIP; and the historical expectations and results of radiocarbon dating for the buildings of this period differ mainly by $100-300$ years, with both chronologists and radiocarbonists preferring the lower end of the possible date range. Both arguments support each other with respect to the past, and, more or less, by value. At the same time, they are supported by our findings, which suggest that the Old Kingdom must be shifted back in time by about two centuries ( $2810 \pm 5 \mathrm{BCE}$, if Khufu's pyramid was oriented toward Thuban vs. Shaw's estimate of 2588 BCE).

## VII. The puzzle of Khafre

As seen in the high-resolution azimuth graph (Fig. 4), the data of the pyramid of Khafre do not fit the trend line for the $4^{\text {th }}$ Dynasty pyramids. K. Spence demonstrated that Khafre's azimuth fit the main trend line after changing its sign. To justify the sign reversal, she suggested that the orientation ritual for this pyramid was conducted in the season opposite to the conventional one. As previously discussed, the "balanced" position of Meskhetiu was apparently sacred. No deviation from it must be assumed in the orientation rituals for pyramids. Therefore, it seems unlikely that the builders made room for

[^21]such a deviation from the orientation norm ritually established by switching to the lower culmination of the asterism. The orientation of Khafre's pyramid thus remains unexplained.
W. M. F. Petrie, reported ${ }^{77}$ the following about Khufu's pyramid: the core has an average azimuth of $-05^{\prime} 16^{\prime \prime} \pm 20^{\prime \prime}$; the casing has an average azimuth of $-03^{\prime} 43^{\prime \prime} \pm 12^{\prime \prime}$; the descending passage has an azimuth of $-03^{\prime} 44^{\prime \prime} \pm 10^{\prime \prime}$ for its entire length and an azimuth of $-05^{\prime} 49^{\prime \prime} \pm 7^{\prime \prime}$ for the part built inside the masonry. If these figures are accurate, then given similar azimuths, the construction of the pyramid can be divided into two stages: 1) construction of the core $\left(-05^{\prime} 16^{\prime \prime}\right)$ and the descending passage ( $-05^{\prime} 49^{\prime \prime}$ ) inside the masonry; 2) construction of the casing ( $-03^{\prime} 43^{\prime \prime}$ ) and the adjustment of the resulting azimuth of the descending passage ( $-03^{\prime} 44^{\prime \prime}$ ) by cutting its rock part.

| Pyramid | Azimuth <br> (E side), arcmin. | Azimuth <br> (W side), arcmin. | Azimuth <br> (passage), arcmin. |
| :--- | :---: | :---: | :---: |
| Khufu ( $1^{\text {st }}$ stage) | -5.4 | -5.7 | -5.8 |
| Khufu $\left(2^{\text {nd }}\right.$ stage $)$ | $-4.0(-3.4)$ | $-3.9(-3.7)$ | -3.7 |
| Khafre | $-6.2(-4.0)$ | $-4.4(-4.2)$ | -5.6 |

Table 7. Data on the azimuths of the sides and descending passages of the pyramids of Khufu and Khafre according to Petrie ${ }^{78}$; data in parentheses according to Nell and Ruggles ${ }^{79}$.

It can be seen that not only are the casing sides of these two pyramids oriented identically, but their descending passages also have a strikingly identical alignment. The proposed model does not explain why the data for these two pyramids coincide. However, previously proposed hypotheses may help in this regard. They are summarized here followed by a brief commentary:

1) Hypothesis of copying of the alignment. The author D. Rawlins [2003, 3] formulates it as follows: "[...] there was no need to celestially orient Khafre's pyramid independently, since its east

[^22]side (casing) is (deliberately?) almost exactly twice as near the west side of Khufu's Great Pyramid as the Khufu pyramid's $W \& E$ sides are to each other. [fn.3] So, for an ancient Egyptian surveyor, orienting the Khafre pyramid by simple geometry (i.e., non-celestially) from the $N-S$ line of the Khufu pyramid's west side was no harder than internally orienting a side of either pyramid from its own opposite [...]" Copying the alignment of Khufu's west side would have been a serious violation of the prescribed acts of the foundation ceremony, one of which, the "stretching of the cord" rite, involved stellar orientation. If copying were admissible, one would expect Menkaure to copy Khafre's alignment, but this is not the case.
2) Hypothesis of the change of Khufu pyramid's position. The author O. Kruglyakov [2016, 2] formulates it as follows: "During the reign of Khufu and for his burial, stellar orientation and marking of the construction site on a hill was carried out. But after marking the square, maybe even after laying the foundation, the builders changed their minds for some reason, stopped work there, abandoned this site and built a pyramid for Khufu to the north-east, where we see it today. And only after the death of Djedefre, with the reign of Khafre, a pyramid was erected for him on that long-abandoned foundation." Azimuth data testify in favor of the marking up of both pyramids very close in time (Thuban's $-5^{\prime}$ epoch), so the change of plans should have occurred at the earliest stages of work. Within the proposed hypothesis, two key questions need to be answered: a) what significant reason could have forced Khufu to move from the gentle part of the hill to a new site perilously close to the steep northeast terrace? b) why Khafre's architects could use the old Khufu's markup although it has already lost its relevance ${ }^{80}$ ?
3) Hypothesis of Khufu's double project. The authors M. Shaltout, J. Belmonte and M. Fekri [2007b, 417-419] formulate it as follows: "[...] the Sphinx and the two large pyramids, the
${ }^{80}$ Ignoring an error that exceeded $12^{\prime}$ is strange in light of the fact that the azimuth of Khufu's casing has been corrected by an insignificant $2^{\prime}$ (see Table 7).
associated temples and the large necropolis for the other members of the royal family may have formed part of a single master plan to reproduce on Earth the name of the funerary complex of Queops, Akhet Khufu, the Horizon of Khufu [the authors mean the implementation in the monumental architecture of the N27 sign - the sun disk between two mountains (= pyramids)]. Presumably Khufu was unable to finish such a huge project during his reign of some 23 years, and the unfinished, or perhaps even merely outlined, second pyramid of the group might have been "usurped" and finished by his son Khafre a few years later [...]" Further study by G. Magli [2016] presented a list of clues supporting this hypothesis, while Shaltout's et al. basic idea was never valid ${ }^{81}$. Magli raised some interesting questions, but a detailed review of them is beyond the scope of the current study. This option seems to have an advantage over the others, but the issue needs to be reassessed.

## VIII. Conclusions

A comparison of the azimuth data of the pyramids and data on the precessional drift of circumpolar stars in the sacred "balanced" position of the Meskhetiu (Big Dipper) asterism led to the discovery of a comprehensive pattern that explains the orientation of twelve Old Kingdom pyramids from Djoser to Unas. The discovery of trends with similar gradients in the pyramids' azimuth data indicates that the monuments were oriented toward different stars in the same position of the sky. This find demonstrates that the "imperishable stars" were the goals themselves, thereby refuting the commonly held belief about the orientation of the $4^{\text {th }}$ Dynasty pyramids to the cardinal points, which arose due to the proximity of the direction to true north and the direction to one of the stars (the pole star) chosen by the Egyptians to orient the pyramids.

The persuasive regularity discovered permits the conclusion that the age of the Old Kingdom in the conventional Egyptian chronology

[^23]Multi-star target model for astronomical orientation of the Old Kingdom... has been underestimated by more than two centuries, or, more accurately stated, $222 \pm 5$ years older than Shaw's estimates. An analysis of the results of radiocarbon dating, and a comparison of reconstructions of the Egyptian chronology shows that older dates of the Old Kingdom are more consistent with the ancient chronological sources, and with radiocarbon-determined ages of the monuments from this period. It is important to note that radiocarbon data indicate the construction of the two big pyramids of Giza during a unique astronomical event - the closest approach to the celestial Pole of the Old Kingdom pole star, Thuban ( $\alpha$ Dra). The proposed orientation method is straight-forward, and comports well with what is known about the astronomical knowledge and abilities of the Egyptians at that time. Thus, it is not necessary to invoke a sophisticated, as yet undiscovered method to orient the foundation of Khufu's pyramid relative to the cardinal points, since its remarkably accurate orientation is only a consequence of the special properties of the chosen reference star, in this case the pole star.

The hypothesis about the orientation of the Old Kingdom pyramids toward selected circumpolar stars in the sacred position of Meskhetiu still needs to be verified by more azimuth data, and therefore only future, accurate examinations of pyramids' orientations will confirm or refute the conclusions drawn. The "puzzle of Khafre", lacking an explanation from within the framework of the proposed model, requires further investigation.

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## ABBREVIATIONS

$B d E ́ \quad$ Bibliothèque d'Étude, Institut français d'archéologie orientale. Le Caire.
BIFAO Bulletin de l'Institut Français d'Archéologie Orientale. Cairo.
GM Göttinger Miszellen. Beiträge zur ägyptologischen Diskussion. Göttingen.
JARCE Journal of the American Research Center in Egypt. Boston, Princeton, New York, and Cairo.
JEA Journal of Egyptian Archaeology. London.
$J H A \quad$ Journal for the History of Astronomy.
JNES Journal of Near Eastern Studies. Chicago.
MDAIK Mitteilungen des Deutschen Archäologischen Instituts, Abteilung Kairo. Mainz, Cairo, Berlin, and Wiesbaden.
MMAF Mémoires publiés par les membres de la Mission archéologique française au Caire. Paris.
$Z \ddot{A} S \quad$ Zeitschrift für ägyptische Sprache und Altertumskunde. Berlin and Leipzig.

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SUPPLEMENTARY MATERIALS


Fig. SM1. Picture of the northern sky in the Ramesseum (19 ${ }^{\text {th }}$ Dynasty). (After [Wilkinson 1991, Fig. 3]).


Fig. SM2. Part of the northern panel on the ceiling in the tomb of Ramses VI (20 ${ }^{\text {th }}$ Dynasty). (After [Thuault 2020, Fig. 9]).

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Fig. SM3. Picture of the northern sky in the tomb of Pediamenopet $\left(26^{\text {th }}\right.$ Dynasty). (After [Wilkinson 1991, Fig. 4]).


Fig. SM4. Possible usage of two poles and a looped cord to accurately identify the "balanced" position of Meskhetiu (horizontal alignment of Dubhe-Alkaid) in the "stretching the cord" ceremony. The poles should be set at the same distance from the observer so that Thuban (the pole star), marked with an arrow, is about halfway between them. The cord must be installed horizontally. The observer sits (therefore no tall poles are needed), looking up from below, and waits for the simultaneous appearance of two selected stars in the "viewing gap", formed by two parallel threads of the cord. The width of the "viewing gap" can be adjusted by changing the thickness of the poles and cord and the distance from the observer to the instrument. (Adapted from Stellarium 0.22.1).


Fig. SM5. Synchronism for Beckerath's [1997] high chronology. The Stadelmann's proportion for the duration of construction of the three pyramids of Sneferu are adapted to 35 years of his reign. The pyramid trend lines have slightly smaller gradients ( $+18^{\prime \prime} /$ year for the main trend line; $+18^{\prime \prime} /$ year for the second trend line; $+20^{\prime \prime} /$ year for the third trend line) than in Shaw's chronology, due to the 7 years between reigns of Khafre and Menkaure attributed to Baka (Bikheris), and the longer reigns of Neferefre and Nyuserre (see Table SM2). The interval between two intersections of the pyramid trend lines with the x -axis is 165 years, which also best fits the Thuban-Alioth pair from Table 3. To get a match the pyramids' data have been shifted by 208 years into the past.

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Fig. SM6. Synchronism for the chronology of Hornung, Krauss and Warburton [2006]. The Stadelmann's proportion for the duration of construction of the three pyramids of Sneferu are adapted to 34 years of his reign. The pyramid trend lines have slightly greater gradients $\left(+20^{\prime \prime} /\right.$ year for the main trend line; $+24^{\prime \prime} /$ year for the second trend line; $+24^{\prime \prime} /$ year for the third trend line) than for Shaw's chronology, due to the ultra-short reign of Menkaure (6 years) and the shorter reign of Neferirkare (see Table SM2). The interval between two intersections of the pyramid trend lines with the $x$-axis is 128 years, which also best fits the Thuban-Alioth pair from Table 3. To get a match the pyramids' data have been shifted by 300 years into the past.

| Pyramid | $\begin{array}{\|c\|} \hline \text { Target } \\ \text { star } \end{array}$ | Start of constr., BCE | Azimuth, arcmin |  |  | Note |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Stellar | Measured | Diff. |  |
| Djoser | Mizar | 2899 | $\begin{gathered} +3^{\circ} \\ 30.3^{\prime} \end{gathered}$ | $+3^{\circ} \pm 30^{\prime}$ | +30.3' | upper boundary of error margin |
| Sekhemkhet | Phecda | 2880 | $-11^{\circ} 5.4^{\prime}$ | $-11^{\circ} \pm 30^{\prime}$ | -5.4' |  |
| Khaba | Megrez | 2872 | $\begin{gathered} -8^{\circ} \\ 30.1^{\prime} \end{gathered}$ | $-8^{\circ} 30^{\prime} \pm 30^{\prime}$ | -0.1' |  |
| Meidum | Thuban | 2845 | -17.4' | $-19.3^{\prime} \pm 1^{\prime}$ | +1.9' |  |
| Bent | Thuban | 2835 | -13.6 ${ }^{\prime}$ | $-14.6^{\prime} \pm 0.3^{\prime}$ | +1.0' |  |
| Red | Thuban | 2826 | $-10.1^{\prime}$ | $-8.7^{\prime} \pm 1^{\prime}$ | -2.4' |  |
| Khufu | Thuban | 2810 | -4' | $-3.6^{\prime} \pm 0.3^{\prime}$ | -0.4' |  |
| Djedefre | Alioth | 2787 | -54.6' | $-47.3^{\prime} \pm 1^{\prime}$ | -7.3' | W side: -50.8' (diff: -3.8') |
| Khafre | Thuban | ? (2779) | ? (+8.5') | $-4.1^{\prime} \pm 0.3^{\prime}$ | ? |  |
| Menkaure | Thuban | 2753 | +18.2' | $+14.1^{\prime} \pm 1^{\prime}$ | +4.1 ${ }^{\prime}$ | $\begin{aligned} & \text { avg. }+18.0^{\prime} \\ & \text { (diff: }+0.2^{\prime} \text { ) } \\ & \text { [Nell, Ruggles } \\ & (2014) \text {, Table } \\ & 6 \mathrm{~b}] \end{aligned}$ |
| Sahure | Alioth | 2708 | -23.8' | $-23^{\prime} \pm 10^{\prime}$ | -0.8 ${ }^{\prime}$ |  |
| Neferirkare | Thuban | 2696 | +40.2' | $+30^{\prime} \pm 10^{\prime}$ | +10.2' | upper boundary of error margin |
| Unas | Alioth | 2596 | +19.9' | $+17.3^{\prime} \pm 0.3^{\prime}$ | +2.6 ${ }^{\prime}$ |  |
| Teti | Phecda | 2566 | $-9^{\circ} 5.3^{\prime}$ | $\begin{gathered} ?\left(-9^{\circ} 15^{\prime} \pm\right. \\ \left.30^{\prime}\right) \end{gathered}$ | ? (+9.7') | azimuth of the pyramid temple |

Table SM1. Deviation of the alignments of the $3^{\text {rd }}-6^{\text {th }}$ Dynasty pyramids from the direction to the reference star in the "balanced" position of Meskhetiu. Data from Sky Charts 4.2.1, taking into account the geographical coordinates of each site.

| King | Dyn. | Reign length, years |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Turin King List | Manetho (Africanus) | $\begin{aligned} & \text { Shaw } \\ & {[2000]} \end{aligned}$ | Beckerath [1997] | Hornung et al. [2006] |
| Nebka | 3 | 19 (Nbk3) | 28 (Nєұє¢оүךऽ) | 19 | 17 | ? |
| Djoser | 3 | 19 (Dsrjt) | 29 (ТоборӨроऽ) | 19 | 20 | 27 |
| Sekhemkhet | 3 | 6 (Dsrtj) | 7 (Tvpcıs) | 8 | 7 | 6 |
| Khaba | 3 | ? | ? | 3 | 24 | 16 |
| Huni | 3 | 24 (Hw..) | ? | 24 |  |  |
| Sneferu | 4 | 24 (Snfrw) | 29 ( $\sum$ ֹ̃ $\rho 1 \zeta$ ) | 24 | 35 | 34 |
| Khufu | 4 | 23 (missing) | 63 ( (ovpis) | 23 | 23 | 27 |
| Djedefre | 4 | 8 (missing) | 25 (PatoıणПऽ) | 8 | 9 | 8 |

Multi－star target model for astronomical orientation of the Old Kingdom．．．

| Khafre | 4 | ？（missing） | 66 （Гоט¢ı¢） | 26 | 26 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Baka | 4 | ？（missing） | 22 （Bıх¢рๆ¢） | － | 7 | 2 |
| Menkaure | 4 | $18 \text { [28?] }$ (missing) |  | 29 | 28 | 6 |
| Shepseskaf | 4 | 4 （missing） | 7 （ $\Sigma \varepsilon \beta \varepsilon \rho \chi \varepsilon \rho \eta \varsigma)$ | 5 | 5 | 6 |
| Userkaf | 5 | 7 （．．k3．．） |  | 7 | 8 | 7 |
| Sahure | 5 | 12 （missing） | 13 （ $\Sigma \varepsilon \varphi \rho \eta \zeta$ ） | 12 | 13 | 13 |
| Neferirkare | 5 | ？（missing） | 20 （Nع¢¢¢¢єрๆ¢） | 20 | 20 | 11 |
| Shepseskare | 5 | 7 （missing） | 7 （ Iøø甲ท¢） | 7 | 7 | 1 |
| Neferefre | 5 | 1 （missing） | 20 （Xعрп¢） | 3 | 11 | 1 |
| Nyuserre | 5 | $\begin{gathered} 11[21 ?]+ \\ (x<4) \\ \text { (missing) } \end{gathered}$ | 44 （Pa日ovpๆऽ） | 24 | 31 | 29 |
| Menkauhor | 5 | 8 （Mnk3hr） |  | 7 | 9 | 8 |
| Djedkare | 5 | 28 （Ddw） | 44 （Tavð¢pๆ¢） | 39 | 38 | 44 |
| Unas | 5 | 30 （Whjs） | 33 （Ovvos） | 30 | 20 | 16 |

Table SM2．Comparison of the reign lengths of the $3^{\text {rd }}-5^{\text {th }}$ Dynasties accor－ ding to ancient sources and reconstructions of Egyptian chronology．

| King | Dyn． | Reign length，years |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Turin <br> King List | Manetho（Afri－ canus） | Manetho （Eusebius） | Beck－ erath ［1997］ | Hor－ <br> nung <br> et al． <br> ［2006］ |
| Narmer |  | ？（Mnji） | 62 （Mnvns） | 60 （Mпv ${ }^{\text {a }}$ ） | 32 | 30 |
| Aha |  | ？（Jt．．） | 57 （A0 $\omega$ Oıs） | 27 （A $\theta \omega \theta \mathrm{l}$ ¢） |  |  |
| Djer | 1 | ？（mis－ sing） | 31 （Kغvкะvך¢） | 39 （Кєvкє－ v̄s） | 47 | 48 |
| Djet | 1 |  | 23 （Oveve¢ๆ¢） | 42 （Oveve－甲ワธ） | 13 | 8 |
| Den | 1 | ？（Kntj） | 20 （Ovo人q＜ıठos） |  | 47 | 43 |
| Adjib | 1 | $\begin{gathered} 74(\mathrm{Mr}- \\ \text { grgph } \end{gathered}$ |  | $\begin{gathered} 26(\mathrm{~N} \varepsilon \mathrm{~s})- \\ \eta \varsigma) \\ \hline \end{gathered}$ | 6 | 8 |
| Semerkhet | 1 | 72 （Smsm） | 18 （इع $¢ \varepsilon \mu \psi \eta \zeta)$ | $\begin{gathered} 18 \text { ( } \sum \varepsilon \mu \varepsilon \mu- \\ \left.\psi \eta)^{2}\right) \end{gathered}$ | 8 | 8 |
| Qa＇a | 1 | 63 （．．bh） | 26 （BıᅱveגๆS） | $\begin{gathered} 26 \text { (OvBicv- } \\ \theta \eta \varsigma) \end{gathered}$ | 25 | 25 |
| Hetepsek－ hemwy | 2 | $\begin{gathered} 95 \\ \text { (..b3w..) } \end{gathered}$ | 38 （BoŋӨos） | －（B由ðos） | 28 | 30 |
| Nebre | 2 | ？（．．．${ }^{\text {c．．．）}}$ |  | －（X $\mathrm{X} 0 \mathrm{\rho}$ ） | 15 |  |
| Ninetjer | 2 | 95 （．．ntr） | 47 （Bıvo日pıs） | －（Biopis） | 43 | 40 |
| Wadjenes | 2 | 70 （．．s） | ？（Thac） | －（－） | 7 |  |
| Senedj | 2 | 54 （Snd ） | 41 （ $\Sigma \varepsilon \theta \varepsilon \vee \eta \zeta)$ | －（－） | 11 | ？ |

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| Peribsen | 2 |  | ? | -(-) | - | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sneferka | 2 | 70 ( 3 k 3$)$ | 17 (Xaıpף¢) | -(-) | 5 | - |
| Neferkasokar | 2 | $\begin{gathered} 8 \\ (N f r k 3 s k r) \\ \hline \end{gathered}$ | 25 (Nє¢¢ $¢ \chi \varepsilon \rho \square \varsigma)$ | - (-) | 8 | - |
| Hudjefa | 2 | 11 ( $h w d f$ ) | 48 ( $\Sigma \varepsilon \sigma \omega \chi \rho 1 ¢)$ | $\begin{gathered} 48\left(\sum \varepsilon \sigma \omega-\right. \\ \chi \rho!\varsigma) \end{gathered}$ | 2 | - |
| Khasekhemwy | 2 | 27 (Bbtj) | 30 (Xعv\& ${ }^{\text {¢ }}$ ) | $\begin{gathered} 30(\mathrm{Xev} \mathrm{\varepsilon-} \\ \text { рп૬) } \end{gathered}$ | 27 | 18 |

Table SM3. Comparison of the reign lengths of the $1^{\text {st }}-2^{\text {nd }}$ Dynasties according to ancient sources and reconstructions of Egyptian chronology. Shaw [2000] gives no data on Early Dynastic reign lengths.

## О. В. Пучков <br> МОДЕЛЬ МУЛЬТИЗІРКОВОЇ ЦІЛІ ДЛЯ АСТРОНОМІЧНОЇ ОРІЄНТАЦІЇ ЄГИПЕТСЬКИХ ПІРАМІД ДАВНЬОГО ЦАРСТВА

В епоху пірамід єгиптяни спорудили деякі з найбільш знакових пам’ятників у світі, але їх метод вирівнювання та точні дати будівництва залишаються предметом суперечок. Ця стаття представляє нові археоастрономічні докази, які пояснюють як нібито нерегулярну орієнтацію пірамід Давнього царства, так і пропонують нове вирішення проблеми датування. Аналіз вирівнювання пірамід, побудованих під час 3-6 династій, показує, що вони були орієнтовані не на північ, як передбачається однією з панівних сучасних моделей, а на видатні зірки в північному циркумполярному регіоні. Чітка закономірність виявляється, коли залежне від часу положення цих зірок порівнюється з орієнтацією ряду пірамід, вирівнювання яких відомо. Шаблон пояснює всі наявні азимутальні дані пірамід від Джосера до Унаса та передбачає більш давні дати будівництва цих споруд із похибкою, що не перевищує п'ять років. Підсумовуючи, вік Давнього царства приблизно на два століття старший, ніж традиційно вважається згідно з наявними текстовими реконструкціями єгипетської хронології. Ці результати є сумісними з радіовуглецевими даними, отриманими зі зразків, зібраних у відомих спорудах Давнього царства, таким чином узгоджуючи археологічні дані з археоастрономічними доказами. Єгипетська хронологія служить стандартом для встановлення регіональних хронологій на всьому стародавньому Близькому Сході III тис. до н. е., отже, переглянута хронологія, заснована на представлених тут висновках, вимагає нового погляду на історичні шкали інших цивілізацій, сучасних Стародавньому Єгипту.

Ключові слова: піраміди Давнього царства, астрономічна орієнтація, Месхетіу, нетлінні зірки, єгипетська хронологія, археоастрономія

Статтяя надійшла до редакйії 29.10.2022


[^0]:    ${ }^{1}$ For a summary of the proposed methods see: [Belmonte 2001, S1-S3; Maravelia 2003, 56-61].
    ${ }^{2}$ Edwards' method is essentially a variant of the elongation method, when instead of elongations, the rising and setting positions of a circumpolar star on an artificial horizon is observed.

[^1]:    ${ }^{3}$ Dash [2015, 6] and Lightbody [2020, 45] attribute the observed trend in azimuth data to the improvement in orientation accuracy (from the pyramid of Meidum to the pyramid of Khufu) when using one of the "true north" methods. Indeed, in these cases, a more accurate orientation corresponds to a higher quality construction. However, expected random orientation errors of any of the "true north" methods would have caused the data to be spread evenly on both sides relative to the x -axis in Fig. 1. Moreover, the azimuths of two trend-forming post-Khufu pyramids change sign to confirm the trend.

    4 "All plausible methods of stellar orientation involve establishing the direction of true north, either through bisecting observed positions of a circumpolar or near-circumpolar star, or through alignments directly to circumpolar stars. [...] Establishing north must thus be considered the goal of the act of celestial alignment, regardless of the method used [...]" [Spence 2010, 173].

[^2]:    ${ }^{5}$ There is a hypothesis that the Djoser and Sekhemkhet complexes were oriented differently due to the topographical features of the site [Maragioglio, Rinaldi 1963, 2; Spence 2010, 173]. If in the case of Sekhemkhet, one could assume that the proximity to the burial complex of the predecessor was the decisive factor, and the orientation of the Sekhemkhet complex was not important and therefore caused by the shape of the underlying hill, then in the case of Khaba, this makes no sense: the builders who decided to leave Saqqara could have chosen any suitable site to orient the complex at will. It seems more likely that the choice of certain sites was driven by their suitability for the required orientation, rather than dictated by the terrain.

[^3]:    ${ }^{6}$ See: [Žába 1953, 11; Lauer 1960, 99; Romer 2007, 279; Nell, Ruggles 2014, 329, Table 7]. All the listed sources give the azimuth value of $+3^{\circ}$ ( $+180^{\prime}$ ).
    ${ }^{7}$ See: [Lauer 1960a, 99; Lauer 1962, 183; Romer 2007, 279; Spence 2010, 172; Nell, Ruggles 2014, 329, Table 7]. All the listed sources except of the second give the azimuth value of $-11^{\circ}\left(-660^{\prime}\right)$. The second source gives $-11.5^{\circ}$.
    ${ }^{8}$ See: [Lehner 1996, 510; Romer 2007, 279; Nell, Ruggles 2014, 329, Table 7]. All the listed sources give the range of values from $-9^{\circ}$ to $-8^{\circ}\left(-510^{\prime}\right.$ on average).

[^4]:    ${ }^{9}$ See: [Petrie 1892, 6 (sides), 11 (passage)].
    ${ }^{10}$ See: [Dorner 1986, 51 (sides), 52 (passage)].
    ${ }^{11}$ See: [Dorner 1998, 23 (E side), 27 (passage)].
    ${ }^{12}$ See: [Nell, Ruggles 2014, 316, Table $1 b$ (sides); Petrie 1883, 58 (passage)].
    ${ }^{13}$ See: [Mathieu 2001, 458 (sides), 459 (passage)]. The azimuth of the descending passage cannot be accurately measured due to the absence of corridor blocks in the pit.
    ${ }^{14}$ See: [Nell, Ruggles 2014, 322, Table $3 b$ (sides); Petrie 1883, 104 (passage)].
    ${ }^{15}$ See: [Petrie 1883, 111 (sides), 117 (passage)].
    ${ }^{16}$ See: [Spence 2000, 320, Table 1]. Lightbody [2020, 45] gives azimuth of $-20^{\prime}$ referring to Krejčí.
    ${ }^{17}$ See: [Žába 1953, 11; Spence 2000, 320, Table 1].
    ${ }^{18}$ See: [Dorner 1981, 83].

[^5]:    ${ }^{19}$ Only "precession-susceptible" methods can explain azimuth trend, while Haack's method does not provide the required accuracy, due to the difficulty of observing stars near the horizon [Belmonte 2001, S3].
    ${ }^{20}$ Some researchers [Krauss 2006a, 110-112; Dash 2013, 13] claim that there is no trend in the descending passages' data, since the Bent and Red pyramids, the passages of which are misaligned with the bases (Table 1), deviate from it. The other accurately measured pyramids (Khufu, Khafre, Menkaure, Meidum) have descending passages co-aligned with their bases, hence the misalignment in two Sneferu's pyramids could not be the goal of the builders, and may indicate either their mistake (Bent and Red pyramids were built successively), or a survey error (both values refer to Dorner's surveys). The assumption that the orientation of the passages of these two pyramids reflect the plan of the builders, while the bases are mistakenly rotated by almost the same value ( $-9^{\prime}$ and $-12^{\prime}$ respectively), is unlikely, since in this case the sides' data would "accidentally" confirm the trend given by the other pyramids.

[^6]:    ${ }^{21}$ Only reign lengths are important for computations. Further conclusions are valid for any chronology option that does not deviate much from the Turin King List data on the Old Kingdom.
    ${ }^{22}$ There are different estimates of the length of Sneferu's reign: a) 24 years ([Shaw 2000, 482]; Turin King List); b) 31 years ([Krauss 1996, 43-50]; different arguments); c) 34-35 years ([Verner 2006, 125]: 34 years; [Beckerath 1997, 156-158]: 35 years; documented dates); d) 40 years ([Monnier 2018, 15-18]; volume of construction work); e) 47 years ([Gundacker 2006, 1-373]; extensive analysis); f) 47-48 years ([Stadelmann 1986, 229-240]; volume of construction work). It seems impossible to build all Sneferu's pyramids in 24 years due to the gigantic volume of construction work, so the current author is forced to abandon Shaw's figure in favor of "neutral" middle estimates (34-35 years). In fact, any of the above estimates is acceptable since this value only slightly affects the gradient of the main trend line.
    ${ }^{23} 14$ years (pyramid of Meidum) / 11 years (Bent pyramid) / 22-23 years (Red pyramid) [Stadelmann 1986, 238].

[^7]:    ${ }^{24}$ This value can be estimated as the difference between the expected date of laying Khufu pyramid's base (Table 2) and the date corresponding to the intersection point of the trend line with the x-axis in Fig. 1.

[^8]:    ${ }^{25}$ Since the reference object was close to the meridian during the $4^{\text {th }} \mathrm{Dy}$ nasty (crossed it during Khufu's reign), and the meridian was inside the gray sectors in Fig. 3, then, only objects from these sectors are suitable.

[^9]:    ${ }^{27}$ For details about the "stretching of the cord" ceremony see [Montet 1964; Weinstein 1973].
    ${ }^{28}$ The Palermo Stone mentions the foundation ceremony presumably during the reign of Den, a king of the $1^{\text {st }}$ Dynasty [Wilkinson 2000, 111113]. The earliest known depiction of the ceremony was found on a granite doorjamb, presumably dating back to the reign of Khasekhemwy from the $2^{\text {nd }}$ Dynasty [Engelbach 1934, Pl. XXIV]. Several fragmentary images belong to the Old Kingdom: fragments found in Sneferu's valley temple at Dahshur [Fakhry 1961, 97, Fig. 91]; a fragment from Nyuserre's sun temple at Abu Gorab [Borchardt 1900, 97, Pl. 5].

[^10]:    48 "When an Egyptian with astronomical knowledge, 4000 years ago, turned his "face to the course of the stars" and let his "eye enter into the constellation of the Thigh [Meskhetiu as a bull's foreleg]" - high above the Pole, not low near the horizon - to the right of the north point of the horizon a deity was seen striding to the left, leaning on an object that tapered to $a$

[^11]:    ${ }^{51}$ See also [Erman, Grapow 1926 (Wb. I), 234, 6].

[^12]:    ${ }^{52}$ Some features of the "stretching of the cord" rite suggest that the two poles and the looped cord were not intended to fix the axis or mark the corners of the building under construction (see [Borchardt 1937, 13-14; Žába 1953, 61-62; Isler 1989, 203-205]), but to create an artificial horizon by placing the cord on the poles horizontally. Moreover, the looped shape of the cord could help the observer to accurately identify the horizontal alignment of two stars when they simultaneously appeared in the "viewing gap" formed by the two parallel threads of the cord (see Suppl. Materials, Fig. SM4). For details, see [Puchkov 2019, 6-9].
    ${ }^{53}$ We must also pay attention to the apparent contradiction: the sacred position of the sky, if it existed, was of great importance and should have been mentioned in many sources, but our reconstruction is based only on two Ptolemaic passages and a few drawings from the Middle and New Kingdoms. First, the existence of a special position of the sky is based on evidence (two parallel trends in the azimuth data) and not on assumption. Secondly, a certain ("accurate", "straight", "equal") position of Meskhetiu is mentioned in the descriptions of the orientation ceremony themselves. Thirdly, the Egyptian culture was so conservative that some rituals lasted for about two thousand years with minor changes, so even much later sources may contain useful information. Fourthly, a small number of sources cannot be considered as evidence of the absence of the sacred position of the sky, since very little information has come down to us about many important aspects of funerary rites: for example, if it were not for the above-mentioned Ptolemaic passages, we would have no idea what the Egyptians observed during the orientation rite, despite the fact that it was used for most of the history of dynastic Egypt.

[^13]:    ${ }^{54}$ Comparison of the accuracy of astronomical programs [De Lorenzis, Orofino 2018] testifies to the best accuracy of Sky Charts for such distant epochs as the Old Kingdom (especially regarding the proper motion of stars).
    ${ }^{55}$ For a detailed reasoning of why Thuban is the best reference star for the $4^{\text {th }}$ Dynasty pyramids, see [Puchkov 2019, 21-25].

[^14]:    ${ }^{59}$ There is no evidence that the Old Kingdom Egyptians were interested in true north or the celestial Pole, but we know from the Pyramid Texts that they deified the "imperishable stars" in the circumpolar region [Faulkner 1966, 155-157], so their desire to align structures with these stars is natural. "It thus seems very likely that the decision to use these [circumpolar] stars for the purposes of aligning burial monuments is closely associated with their important role within mortuary beliefs as a model for eternal existence and the location of the king's afterlife (Žába 1953, 20-23)" [Spence 2010, 174].

[^15]:    ${ }^{60}$ The relative dates of the Old Kingdom are known more or less accurately, therefore, one can infer from the gradient of the azimuth trends, and at the same time move the Old Kingdom along the absolute time-scale, since it is not pinned to absolute dates.

[^16]:    ${ }^{65}$ "The extensive prosopographic data from the FIP led Brovarski and Spanel to conclude that a succession of several generations of local administrators held office in many UE towns between the end of the OK and the beginning of Dyn. 11, thus clearly favoring a long model for the period in perfect accord with the data of Manetho. [...] As was argued by Ward and Seidlmayer, the large number of burials in Upper Egyptian cemeteries which are to be dated to the earlier part of the FIP, as well as the fundamental morphological change which can be discerned in the archaeological material exactly in this phase, argue for a period of several generations. Therefore, substantial evidence seems to support Manetho's figure for the length of the Herakleopolitan period" [Seidlmayer 2006, 167].
    ${ }^{66}$ The stela of the Governor of Byblos Yantin indicates that Neferhotep I ( $13^{\text {th }}$ Dynasty) was a contemporary of the kings Zimri-Lim of Mari and Hammurabi of Babylon [Smith 1965, 16-17]. The primary but questionable synchronism for the OK is the alabaster lid of a jar bearing the name of Pepi I, found in the level IIB1 of the Palace G at Ebla, Syria. Two fragments of diorite vessels with the name of Khafre were found in the same level [Sowada $2009,141-145,222-223]$. There is no consensus on when the Palace G was destroyed (estimated range is $2400-2300$ BCE). This synchronism can only limit the lower estimate of Pepi I's reign by the dates of destruction, since the artifact bearing his name may have been as antique as the artifact bearing Khafre's name, when the palace fell.
    ${ }^{67}$ Habicht et al. [2015, 41-50] claim the discovery of the earliest known Sothic date in an inscription on a small jar. Due to the absence of the king's name in the text, the researchers date it stylistically and attribute it to the $5^{\text {th }}$ or the early $6^{\text {th }}$ Dynasty. The uncertainty in the age of the artifact of more than 150 years and the peculiarities of stylistic dating, however, do not make it possible to confidently use this Sothic date. Two years later, Gautschy et al. [2017] combines found Sothic date with $w 3 g$ feast date from Neferefre's funerary temple at Abusir and proposes a new astronomically based chronology. In the $w 3 g$ date from "Document IV" the season name is lost. It is reconstructed by the authors as III [3ht] 28 or III [prt] 28, producing their high

[^17]:    ${ }^{69}$ See: [Bonani et al. 2001, 1302-1311, Appendix 1: Radiocarbon dates].
    ${ }^{70}$ See: [Dee et al. 2009, 1067, Table 2].

[^18]:    ${ }^{71}$ See: [Bronk Ramsey's et al. 2010, Supporting Online Material, Table S1] for the details on samples.
    ${ }^{72}$ The Oxford model (see Table S1) takes into account only 17 OK dates, of which 4 dates are marked as outliers: 11 for Djoser [/Khasekhemwy] ( 2 outliers), 2 for Sneferu, 3 for the late OK (1 outlier) and 1 "extra" date (1 outlier). For comparison: Bonani et al. [2001] used 245 OK dates.

[^19]:    ${ }^{73}$ All Djoser's younger samples from Bonani et al. were collected in the same location and might be originated from later ritual (temple [field nr.]: ARSE68b=ARSE69) or restoration (pyramid [field nr.]: $252=253=258$ ) activity.
    ${ }^{74}$ An Oxford/Vienna interlaboratory comparison [Bronk Ramsey et al. 2010, SOM, Table S3] shows that results differ by $40-50{ }^{14} \mathrm{C}$ years half the time, which can be a problem for small datasets (such as the OK dataset by Oxford). This comparison does not apply to any of the OK samples for unknown reasons.

[^20]:    ${ }^{75}$ Djoser: ETH-[13652, 13653, 13654, 13658, 13659]; Sekhemkhet: ETH13750, SMU-1368; Djedefre: DRI-2969, ETH-[13745, 13745a, 13746, 13747], SMU-[1357, 1356]; Shepseskaf: ETH-[13729, 13729a, 13729b]; Teti: ETH-[13638, 13639, 13640, 13541, 13542, 13543]. Outliers and single samples for each site are excluded.

[^21]:    ${ }^{76}$ Dee et al. [2013] created a model for the $1^{\text {st }}$ dynasty that corresponds to the conventional dates. Two points are important: 1) these data, as well as the OK data, are variable (all dates for Aha [Hd-12947, Hd-12926], Djet [OxA-26835], part of the dates for Djer [OxA-23195, OxA-26824, OxA-26826, OxA-27253], single dates for Den [Hd-12952] and Qa'a [OxA-27250] support high chronology, the rest (22) - conventional low chronology; reasons for the variability need to be investigated); 2) these data do not allow us to confidently calculate the age of the OK, since the chronological data on the Early Dynastic Period are contradictory (see Table SM3) and therefore the duration of the $2^{\text {nd }}$ Dynasty varies significantly according to different authors (204 years [Shaw 2001, 482]; 146 years [Beckerath 1997, 187]; 140 years [Hornung et al. 2006, 490]; 225 years [Grimal 1992, 389]).

[^22]:    ${ }^{77}$ See: [Petrie 1883, 38-39 (sides), 58 (passage)].
    ${ }^{78}$ See: [Petrie 1883, 38-39 (Khufu's sides), 58 (Khufu's passage), 97 (Khafre's sides), 104 (Khafre's passage)].
    ${ }^{79}$ See: [Nell, Ruggles 2014, 316, Table $1 b$ (Khufu), 322, Table $3 b$ (Khafre)].

[^23]:    ${ }^{81}$ "Khufu's pyramid was Akhet Khufu. Akhet is written with the crested ibis [G25] and elliptical land [N18] sign, not with the hieroglyph of the sun disk between two mountains [N27]." [Lehner 1997, 29]. See [Tedder 2007] for details.

